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The dynamic multiperiod vehicle routing problem with probabilistic information

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ABSTRACT

This paper introduces the Dynamic Multiperiod Vehicle Routing Problem with Probabilistic Information, an extension of the Dynamic Multiperiod Vehicle Routing Problem in which, at each time period, the set of customers requiring a service in later time periods is unknown, but its probability distribution is available. Requests for service must be satisfied within a given time window that comprises several time periods of the planning horizon. We propose an adaptive service policy that aims at estimating the best time period to serve each request within its associated time window in order to reduce distribution costs. The effectiveness of this policy is compared with that of two alternative basic policies through a series of computational experiments.

1. Introduction

The purpose of this paper is to model and solve the Dynamic Multiperiod Vehicle Routing Problem with Probabilistic Information (DVRPP). In this capacitated routing problem, calls for service arrive throughout a discrete time horizon and must be fulfilled within a time window that comprises several time periods. In addition to known information about customers (request time windows and distances), it is assumed that at each time period, probabilistic information regarding calls for service in future time periods is available. At each time period, two decisions must be made: (i) the choice of the subset of pending service requests to satisfy and (ii) the design of the vehicle routes.

The DVRPP belongs to the classes of both multiperiod and stochastic vehicle routing problems. After the pioneering work of Wilson and Colvin [20], a great deal of research has focused on dynamic vehicle routing problems. A recent review of applications and solution methods is given in [17]. Several researchers have investigated problems in which information on travel times, customer demands, customer locations or server availabilities evolves over the planning horizon. Nevertheless, most available studies tackle situations where requests for service arrive dynamically, as it is the case of this work. In addition, several authors have considered different degrees of dynamism for the frequency at which new data becomes available and have investigated the impact of new data on the solution [13].

In the DVRPP service routes are designed at each time period, while most previous work on dynamic vehicle routing considers situations in which predefined routes are dynamically modified to account for new information. For example, Bertsimas and Van Ryzin [7] study a routing problem in which vehicle routes are adapted to include customers as their requests for service arrive. Customers are uniformly located in the plane and service requests arrive according to a Poisson process. They identify alternative optimal policies depending on whether the traffic is heavy or light. This work was extended by Papastavrou [16] who presented a new policy with good performance, independently of traffic density. Similarly, Ghiani et al. [10] designed routes that visit all the known customers and in which it is easy to insert late-call customers. In the same spirit, Moretti Branchini et al. [15] presented and compared some adaptive and distributed algorithms for routing a group of vehicles through a set of customers having stochastic locations and requirements.

Few works have proposed models in which the set of pending customers in a given time period will depend both on the vehicle routes performed in earlier time periods and on new service requests. An example is the Dynamic Multiperiod Vehicle Routing Problem (DMVRP) introduced in [1] and later generalized in [19]. Angelelli et al. [1] consider a multiperiod vehicle routing problem in which, at each time period, some customers make a service request that has to be fulfilled during one of the next two time periods. Once the set of customers to serve at a given time

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period is decided, these are served through optimal routes by uncapacitated vehicles. The aim of the problem is to minimize the total travel cost over the planning horizon. In Wen et al. [19] have extended this problem in two ways. On the one hand, vehicle capacities are taken into account. On the other hand, the maximum number of days before fulfilling a request may differ among customers. Moreover, two additional objective functions are considered, which are related to the workload balance over the planning horizon and to the number of pending requests within their feasible time interval. The authors propose a rolling-horizon procedure in which the service day chosen for each pending customer depends on its distance to other customers with common feasible service days. In both papers, at each time period, decisions have to be made using only information on service requests that have already been placed.

Even in the context of dynamic problems where solutions are gradually adapted to evolving data, probabilistic knowledge of future events can be useful to reduce costs or to improve the solutions with respect to other criteria [12]. Following this idea, in this paper we extend the DMVRP by considering that probabilistic information on future service requests is available at each time period and for each customer without a pending order. That is, at any time period, we know the probability of demand for subsequent time periods for each non-pending customer. Our aim is to use this information to improve the distribution costs. Hence, at each time period a set of routes are designed to serve a subset of the pending customers, while service to the remaining pending customers is postponed to later time periods. Broadly speaking, the DMVRP is the deterministic counterpart of the DVRPP.

Two basic policies can be applied in this context: to serve each pending customer at the beginning of its time window (Early Policy, EP), or to serve each pending customer at the end of its time window (Delayed Policy, DP). There are, however, two main reasons for deviating from such policies. First, the basic policies can lead to infeasible vehicle routing subproblems at some time periods, since vehicles are capacitated and the fleet size is limited. Second, distribution cost savings would be obtained if neighboring customers were scheduled at the same time period. From this point of view, at any time period, the attractiveness of pending customers whose time window is not yet closing will depend on their location with respect to other customers who have to be served urgently, and on the likelihood of receiving requests of neighboring customers in the near future.

According to this idea, the adaptive policy proposed in this paper is based on defining at each time period a vehicle routing subproblem in which not all the pending customers need to be served. Instead, profits are assigned to pending customers, and a Prize Collecting VRP (PCVRP) is then solved to simultaneously define the set of pending customers who will actually be served, as well as the vehicle routes. These profits, which use a compatibility index based on the geometric distribution of customers, capture the willingness to serve each pending customer and depend on the urgency to serve it (measured by the remaining time within its time window) as well as on the probability that other neighboring customers will require service during its time window.

The remainder of this paper is organized as follows. A formal description of the DVRPP is presented in the next section, together with the notation. In Section 3 we describe the proposed solution algorithm and the subproblems needing to be solved at each time period. As mentioned in [6], due to the dynamic nature of the problem, a solution to the DVRPP cannot be a static output but is instead a solution strategy. For this reason, to assess the effectiveness of the proposed solution method, its output is compared with the solutions resulting from applying the two alternative basic policies. The results of these comparisons are presented and analyzed in Section 4. Conclusions are summarized in the last section.

2. Problem definition and notation

We consider an undirected graph $G=(V,E)$, where $V=\{0,\dots,n\}$ is the vertex set and $E=\{(i,j): i,j\in V, i<j\}$ is the edge set. A depot is located at vertex 0 and the remaining vertices are customers. The distance between two vertices $i,j\in V$ is denoted by c_{ij}. An homogeneous fleet of vehicles of capacity Q is located at vertex 0. These vehicles are indexed in a set K. We denote by q_i the demand of customer $i\in V\setminus\{0\}$, which we assume to be fixed and known in advance. Furthermore, there is a time horizon T, finite or infinite, and service requests of the customers must be satisfied within fixed time windows made up of several consecutive time periods. At each time period $t\in T$, the customers are partitioned into the following two sets:

- V^t is the set of pending customers at t, i.e. those with an active request of service. The service request of customer $i\in V^t$, must be satisfied within its time window $[l_i,u_i]$. Since t is the first time period when the request of pending customer $i\in V^t$ can be satisfied, we assume that $l_i=t$.
- $V\setminus V^t$ is the set of customers with no pending service request at t.

At time period $t\in T$, for each $\ell\ge t$, we denote by $p^t_{i\ell}$ the probability that the next service time window of customer i contains time period ℓ. Therefore, for $i\in V^t$, $p^t_{i\ell}=1$, if $\ell\in[l_i,u_i]$, and $p^t_{i\ell}=0$ otherwise. For each customer $i\in V\setminus V^t$ we assume that the probabilities $p^t_{i\ell}$ are known.

The DVRPP is to find a set of minimum cost routes for each time period in the planning horizon such that:

- All routes start and end at the depot.
- The service requests of all visited customers are satisfied within their time window.
- At each time period, the demand of a customer is either served fully or not at all.
- The vehicle capacities are not exceeded.

Note that although vehicle fixed costs are not explicitly considered here, they can be taken into account by appropriately increasing the costs of the arcs incident to the depot.

The DVRPP is a dynamic problem since at each time period the available information on future service requests is only probabilistic, and the set of customers who need to be considered at each time period depends on the routes already built in previous time periods, and also on the service requests that have arrived after these routes have been fixed.

Observe that when all information concerning service requests is known a priori, the resulting problem becomes a special case of the periodic vehicle routing problem [9] with frequency one. When no information is available about future calls for service, the problem becomes dynamic, and it is no longer possible to identify an optimal static solution; what is optimal for a set of time periods can become suboptimal if an extra time period is added. Moreover, if the data concerning future calls are not fully available, but are only known in a probabilistic sense, the problem then also becomes stochastic.

3. Algorithm

In order to develop a solution policy for the DVRPP, it is necessary to define a rule for determining, at each time period $t\in T$, (i) the set of pending customers to serve at time period t and, (ii) the associated vehicle routes. As mentioned in Section 1, two elementary policies can be considered: EP and DP. Whereas both