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# A maritime inventory routing problem: Practical approach

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ABSTRACT

Despite of the practicality of the motivation of the inventory routing problem (IRP), there are few successful implementation stories of IRP based decision support systems which utilize optimization algorithms. Besides the fact that the IRP is an extremely challenging optimization problem, simplifications and assumptions made in the definition of typical IRP in the literature make it even more difficult to take advantage of the developed technologies for IRP in practice. This paper introduces a flexible modeling framework for IRP, which can accommodate various practical features. A simple algorithmic framework of an optimization based heuristic method is also proposed. A case study on a practical maritime inventory routing problem (MIRP) shows that the proposed modeling and algorithmic framework is flexible and effective enough to be a choice of model and solution method for practical inventory routing problems.



## 1. Introduction

When vendor managed inventory re-supply is employed, the opportunity of cost savings in product distribution comes with the freedom of choice of which customers to visit on a given day and how much product to deliver to the selected customers. Since the vendor is responsible for the inventory management for customers, the problem the vendor faces is how to efficiently integrate inventory management with the product distribution. This problem is known as the inventory routing problem (IRP) in the literature, and there exist a fair number of articles studying this problem. A recent review on the inventory routing problem can be found in a section of the review of vehicle routing by Cordeau et al. [8].

A typical IRP concerns the distribution of a single product from a single supply facility to a set of customers using a homogeneous fleet of vehicles. Unlimited product is available at the supply facility, and each customer has its own storage capacity and consumption rate. The IRP tries to minimize total transportation cost over a given planning horizon while guaranteeing no stockouts at any of the customers. It is usually assumed that the length of each route is short enough to be completed in a single time period (e.g. one day). Therefore, for each day, the optimization makes decisions regarding the selection of customers to visit, amount of product delivered to each selected customer, and the routes of vehicles to perform the deliveries. Even with the simplifications and assumptions made in the definition of the typical IRP, it is well known that the IRP is an extremely challenging optimization problem.

From a practical perspective, there exist many simplifications and assumptions made in this definition of the typical IRP: a single supply or production site, unlimited production capacity, unlimited storage capacity at the production site, constant consumption rate over the planning horizon at customers, etc. Since the length of each route is assumed to be short enough to be performed in a single day, the routing decision can be separated from the customer selection decision and the amount of product delivered to the customers. Once the decisions for which customers are visited on a given day and how much product to deliver are made, the remaining elements of the problem form the capacitated vehicle routing problem, which leads to many ideas for solution methods. These assumptions and simplifications in the definition of the IRP impose a substantial limitation in the use of the model for real world applications. Typically the solution approaches are developed such that they take advantage of these characteristics of the problem, which also substantially limits their use in practice.

There exist variations of this typical IRP in order to consider the practical features of real world problems. Savelsbergh and Song [11,12] study a variation of the IRP with the consideration of multiple production sites, storage capacities at the production sites and customer sites, a constant production rate at each production site, and routes longer than a single time unit. Christiansen and Nygreen [6,7], and Christiansen [3] study a maritime inventory routing problem, which involves multiple production ports with constant production rates and limited storage capacities, multiple loads and discharges, and the consideration of travel time in the inventory update constraints due to long voyages. Most of these variations are very application oriented. Therefore, the solution methods tend to take advantage of the problem specification such as constant production and consumption rates for the entire planning horizon. This is a reasonable approach since their goal is to develop a solution

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method for their particular problem of interest. As pointed out by Andersson et al. [2], unlike other routing problems with clear definitions and assumptions, almost every paper concerning combined inventory management and routing has a new version of the problem. Therefore, one of the major difficulties of applying IRP techniques from the literature to a new real world IRP problem is that it is not obvious how to extend these models and solution methods to a more general problem setup, such that they are flexible enough to accommodate various practical features which may appear in other real life inventory routing applications.

When seagoing vessels are responsible for the transportation in IRP, the IRP becomes a maritime inventory routing problem (MIRP). Christiansen and Fagerholt [4] describe a basic MIRP as the transportation of a single product that is produced at loading ports and consumed at discharging ports where each port has a given inventory storage capacity and a production or consumption rate. Christiansen et al. [5] thoroughly review maritime transportation from the operations research perspective. Al-Khayyal and Hwang [1] also describe various aspects of maritime transportation from a modeling point of view. Recently, Furman et al. [9] introduced a MIRP based on a real business application in the petroleum industry. This problem is similar to the problem studied by Christiansen et al. [3,6,7], but the complex cost structure for voyage charter vessels, daily changing production and consumption schedules, and various practical requirements such as draft limits make the problem studied in Furman et al. [9] unique. Demurrage costs are incurred when allowed laytime (i.e. time at port) is exceeded, for example, by waiting to load or to discharge more product. A fixed cost is charged to each leg regardless of the amount of product on board. However, an overage freight cost is applied to any amount of production loaded in excess of a pre-specified cargo amount known in the shipping industry as the part-cargo minimum. This overage cost is charged for the entire voyage. These additional cost components make the cost structure for voyage charter vessels complex. Daily changing production and consumption rates are some of the most important differentiating practical features in the problem because an assumption of constant production and consumption would significantly hamper the operational use of such a model. Draft limits are quite interesting features to marine transportation problems, since a vessel may or may not be allowed to visit a port depending on the weight of the vessel which is affected by the amount of product it carries. Various other practical features can also be introduced into the problem. The problem studied in this paper fits nicely as a test case for the practicality of the proposed modeling and algorithmic framework due to its inclusion of several differentiating characteristics not typically seen in the literature. The problem studied in this paper is the same MIRP discussed by Furman et al. [9]. Through the study of this practical problem, we introduce a flexible modeling framework and a simple optimization based heuristic approach, both of which capture the fundamental features of the MIRP. We also show how various practical features of the MIRP can be effectively handled in the proposed framework. Computational experiments on several realistic test cases illustrate the effectiveness of the proposed method.

The remainder of the paper is organized as follows. In Section 2, the formal problem description is introduced. Section 3 discusses various techniques for the solution methods as well as the development of a large scale neighborhood search procedure as a practical solution technique. Section 4 illustrates how various additional practical features can be incorporated into the proposed model. In Section 5, computational experiments are conducted with the model and algorithm described in this paper. Some comparison of results between the method in this paper and the one in Furman et al. [9] are also presented. Finally, in Section 6 the conclusions are discussed.

## 2. Problem description

The goal of the problem is to find an optimal schedule for routing a heterogeneous pool of seagoing ships in the loading, transporting and discharging of a single bulk product to and from multiple ports while maintaining all capacities, constraints and restrictions related to inventory or to specific ports. This problem is defined as follows.

Given:

- Set of ports at which product is either produced or consumed.
- Set of heterogeneous vessels (some of them may be already chartered).
- Time horizon for planning.
- Daily amount of production or consumption of product at each port.
- Initial inventories of product and inventory limits at each port.
- Load or discharge quantity ranges at each port.
- Travel time between ports.
- Draft limits and other port restrictions.
- Cargo capacity and part cargo minimum for vessels.
- Transportation cost calculation parameters.

The objective is to minimize total transportation cost minus added value by transported product while the routes of specific vessels, the timing of each particular leg of the voyages, and quantities of product loaded and discharged are determined by the solution of the problem such that all constraints are satisfied. This problem already has several complicating characteristics including: flexible cargo sizes, port draft limits, daily changing production and consumption rates, vessels may load and discharge at multiple ports, vessels may revisit ports, limited berth availability at ports, and route-, cargo size- and timing-based transportation costs.

A bulk product is distributed from a set $\mathcal{J}^P$ of production ports to a set $\mathcal{J}^C$ of consumption ports over a planning horizon $T$. The set $\mathcal{J}$ of all ports is the union of $\mathcal{J}^P$ and $\mathcal{J}^C$. A set $\mathcal{J}^D$, a subset of $\mathcal{J}$, represents a set of ports with draft limits. The draft limit restricts the amount of product onboard when a ship enters and leaves the port. The number of loads/discharges by a vessel at port $j$ may be limited. This means that each vessel cannot load/discharge at port $j$ more than $m_j$ times. In maritime transportation, a vessel rarely visits a port more than once within a voyage. However, $m_j > 1$ allows a vessel to load/discharge from/to a port more than the maximum amount of loading/discharging per time unit. When a vessel capacity is relatively larger than the storage capacity at a port, this feature can be quite useful.

The model developed in this paper is a discrete time mixed integer linear programming model in which each time index $t$ belongs to the set $\{1,2,\ldots,T\}$. For practical purposes, the time unit used in the computational experiments is a day. However, different time units can easily be applied depending on necessity. Each port $j \in \mathcal{J}$ has an initial inventory $I_{(j,0)}$ at the beginning of the planning period. Each port $j \in \mathcal{J}$ has also a minimum inventory level $I_{(j,t)}^{\min}$ at time $t$ and a maximum inventory level $I_{(j,t)}^{\max}$ at time $t$ for each $t \in \{1,2,\ldots,T\}$. When a load or a discharge happens at port $j \in \mathcal{J}$, its amount has to be greater than or equal to $f_j^{\min}$ and less than or equal to $f_j^{\max}$. Each port $j \in \mathcal{J}^P$ produces $p_{(j,t)}$ amount of product from time $t-1$ to time $t$, and each port $j \in \mathcal{J}^C$ consumes $d_{(j,t)}$ amount of product from time $t-1$ to time $t$. The travel time between ports $j$ and $j'$ is $t_{jj'}$, and we assume that $t_{jj'}$ is a multiple of the time unit employed. We also assume that travel time includes the time a vessel spends at a port. This port time may consist of berthing time, loading time, discharging time, de-berthing time, and so on.

A set $\mathcal{V}$ of voyage charter vessels is available for transporting the product. A voyage begins when an empty vessel visits a port and