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Tabu search and lower bounds for a combined production–transportation problem

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ABSTRACT

In this paper we consider a combined production–transportation problem, where n jobs have to be processed on a single machine at a production site before they are delivered to a customer. At the production stage, for each job a release date is given; at the transportation stage, job delivery should be completed not later than a given due date. The transportation is done by m identical vehicles with limited capacity. It takes a constant time to deliver a batch of jobs to the customer. The objective is to find a feasible schedule minimizing the maximum lateness.

After formulating the considered problem as a mixed integer linear program, we propose different methods to calculate lower bounds. Then we describe a tabu search algorithm which enumerates promising partial solutions for the production stage. Each partial solution is complemented with an optimal transportation schedule (calculated in polynomial time) achieving a coordinated solution to the combined production–transportation problem. Finally, we present results of computational experiments on randomly generated data.

1. Introduction

Traditional scheduling research deals with problems of sequencing jobs on processing machines without taking into account transportation issues. Recent trends in scheduling involve extended scheduling models where more practical constraints are included. In particular, in a typical supply chain system materials and resources are available at some release dates at the manufacturer's site; the manufacturer should process them in accordance with technological constraints; finally, finished goods should be delivered to a customer by given due dates. In the context of a supply chain, scheduling of production cannot be done in isolation from scheduling of transportation since a coordinated solution to the integrated problem may improve the performance of the whole supply chain.

The first stage of our model – *production* – deals with a single production facility and a set of n jobs, which should be processed one at a time. The jobs may have different processing requirements in terms of the production time and may be available at different release dates. The second stage – *transportation* – deals with the delivery of n finished goods to a customer using m transportation vehicles which have the same delivery characteristics: equal transportation times from the production site to the customer and equal capacities, i.e. the maximum number of jobs which can be transported by a vehicle in one batch. The objective is to define a production and a transportation schedule so that the jobs are delivered to the customer by their due dates. In a more general setting, it is required to minimize the maximum lateness among the jobs, see Section 2 for a formal definition.

The results related to the problem under consideration are scattered among a broad range of publications on (A) *production–transportation* and (B) *generalized flow-shop models* with the second stage involving m identical batching machines.

(A) The literature on production–transportation is vast ranging in multiple parameters. With several survey papers available, e.g., [5,8,13,26], we refer the reader to the most recent review by Chen [6]. The results related to our study fall in the category of "batch delivery by direct shipping" discussed in Section 5.1 of the review. Eliminating the non-relevant models with cost factors (for which it is usually assumed that the number of vehicles is sufficiently large, $m=n$, [11–13,25]), the models with resource availability constraints and those with min-sum criteria, we review here the most closely related papers [16,28] with min–max criteria. Their main outcomes can be summarized as follows. If all jobs are available simultaneously, then the production–transportation problem with the makespan objective is solvable in $O(n \log n)$ time [16], while the following two generalizations are NP-hard: the version studied in [16] with two machines at the production site operating as a flow-shop and the version studied in [28] with an additional transportation stage from a supplier to

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the production site which precedes the introduced production–transportation model. No results are available for the production–transportation model with arbitrary release dates and due dates.

(B): In the flow-shop model with two machines, each job should be processed on the first machine and then on the second one. If the second machine operates in a batching mode (i.e. several jobs can be processed simultaneously) and the batch processing time is constant independent of the jobs included in the batch, the corresponding flow-shop model is equivalent to our production–transportation model with one transportation vehicle. To the best of our knowledge, the flow-shop problem with a batching machine and unequal release dates and due dates was not addressed in the literature. The studied models deal with the following special cases:

- If all jobs have equal release dates and equal due dates, then the corresponding problem is solvable in $O(n \log n)$ time [1]. Note that, as discussed in part (A), an algorithm with the same complexity is also known for the more general case with multiple batching machines (multiple transportation vehicles) [16].
- If the jobs have arbitrary release dates but equal due dates, then the corresponding problem is NP-hard in the strong sense [27] and, as shown in the same paper, it can be solved by a 2-approximation algorithm. Note that the NP-hardness result is also proved in a later paper [22], which uses the production–transportation terminology (A); the same paper presents also a 5/3-approximation algorithm. The most recent approximation algorithm has a worst-case ratio of 3/2 [21].

Summarizing we observe that there is a lack of research addressing the production–transportation problem in its general setting when the jobs have unequal release dates and unequal due dates. Also, as stated in the review paper [6], "the majority of the existing work has been centered on clarifying complexity of some problems, most of which are, unfortunately, NP-hard… Therefore it is worthwhile to design fast heuristics or exact branch-and-bound algorithms for such problems". In this paper, we pursue this line of research by developing a tabu search algorithm for the general case of the production–transportation problem. It is hard to perform a fair comparison of our algorithm with published algorithms since they have been developed for models which differ too much from the one we consider. For example, the heuristics for a flow-shop model with batching machines often deal with batches of unequal size, see, e.g., [17,18,20,23,24]; the production–transportation papers often consider non-identical vehicles [29] and take into account the routing issues [9]. Due to this reason, we also pay special attention to lower bound calculations and estimate the quality of our tabu search algorithm with respect to the calculated lower bound values.

The remainder of this paper is organized as follows. After defining our problem formally and discussing known results for it in Section 2, a mixed integer linear programming formulation is given in Section 3. Section 4 is devoted to different lower bound calculations. In Section 5 a tabu search algorithm is presented. Computational results can be found in Section 6. We conclude the paper with some remarks in Section 7.

2. Problem formulation

In this paper, we consider a combined production–transportation problem, which can be described as follows. There are n jobs of a set $N=\{1,2,\ldots,n\}$ which have to be processed at a production site before being delivered to a customer by transportation vehicles. The corresponding stages are called production and transportation, and the two operations of a job are called production and transportation operations, accordingly.

At the production stage, each job $j \in N$ becomes available for processing at its release date r_j and has to be processed for $p_j \geq 0$ time units. The production site operates as a single machine, i.e. it processes at most one job at any time. Additionally, due dates d_j for the jobs $j \in N$ are given by the customer with the meaning that job j should be delivered not later than time d_j. We assume that all input data are integer.

After job j has finished processing at the production stage, it becomes available for transportation to the customer. The delivery is performed by m identical vehicles with limited capacity where any vehicle can carry no more than b jobs at any time. It takes a constant time τ to deliver a batch of jobs to the customer. Additionally, we assume that the time for returning back from the customer to the production site is negligible. The overall performance of a schedule is measured in terms of the maximum lateness $L_{\max} := \max\{L_j \mid j \in N\}$, where $L_j := C_j - d_j$ is the lateness of job j and C_j denotes the time where the delivery of job j to the customer is completed.

Another objective function equivalent to $L_{\max}$ is the extended makespan $C^q_{\max} = \max\{C_j + q_j\}$ where the jobs have tails q_j instead of due dates d_j. A tail q_j means that after the completion time C_j of job j additionally q_j time units are needed before the job is finished (tails of different jobs can be executed simultaneously). If we set $q_j := D - d_j$ for all $j \in N$ with a constant $D \geq \max_{j \in N} d_j$, it is easy to see that $C^q_j = C_j + q_j = C_j - d_j + D = L_j + D$ holds. Since D is a constant, minimizing $L_{\max}$ is equivalent to minimizing $C^q_{\max}$.

A feasible production–transportation schedule may be completely specified by

- a processing sequence of the jobs on the production machine and
- a sequence of batches for the transportation stage.

From these two sequences a feasible (left-shifted) schedule, in which every operation starts as early as possible, may be calculated as follows. At the production stage, each job can start immediately after it is released and the previous job is completed, i.e. the starting time S^p_j of job j on the production machine is $S^p_j = \max\{r_j, S^p_i + p_i\}$, where i is the job directly processed before j. The completion time C^p_j of j on the production machine is $C^p_j = S^p_j + p_j$.

For the transportation stage we have a partitioning of all jobs into a sequence $(B_1, B_2, \ldots, B_\alpha)$ of batches where $\alpha \in \{\lceil n/b \rceil, \ldots, n\}$ denotes the number of used batches and the sequence of batches determines their starting order. Since the transportation time τ is constant and the objective function is regular, it is sufficient to consider schedules in which the assignment of the batches to the vehicles is done in a cyclic way such that vehicle $\nu \in \{1,\ldots,m\}$ processes batches $B_\nu, B_{\nu+m}, B_{\nu+2m}$, etc. In such a schedule the starting time S_{B_k} of batch B_k can be determined as

$$S_{B_k} = \begin{cases} \max\{C^p_j \mid j \in B_k\}, & 1 \leq k \leq m, \\ \max\{\max\{C^p_j \mid j \in B_k\}, S_{B_{k-m}} + \tau\}, & m < k \leq \alpha. \end{cases} \quad (1)$$

Furthermore, the completion time C_j of each job $j \in B_k$ is given by $C_j = S_{B_k} + \tau$.

Example 1. Consider an instance with $n=5$ jobs, $m=2$ vehicles with capacity $b=2$, and delivery time $\tau=4$. The job characteristics are defined as follows:

j	1	2	3	4	5
r_j	0	0	5	3	5
p_j	1	2	2	1	3
d_j	5	10	24	12	16

With $D = d_3 = 24$ we get the tails $q_1 = 19, q_2 = 14, q_3 = 0, q_4 = 12, q_5 = 8$. Let us assume that on the production machine the jobs are processed in the sequence (1,2,4,3,5); in the transportation stage the batch sequence is ({1},{2,4},{3},{5}). A corresponding