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A knapsack problem as a tool to solve the production planning problem in small foundries

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ABSTRACT

According to recent research carried out in the foundry sector, one of the most important concerns of the industries is to improve their production planning. A foundry production plan involves two dependent stages: (1) determining the alloys to be merged and (2) determining the lots that will be produced. The purpose of this study is to draw up plans of minimum production cost for the lot-sizing problem for small foundries. As suggested in the literature, the proposed heuristic addresses the problem stages in a hierarchical way. Firstly, the alloys are determined and, subsequently, the items that are produced from them. In this study, a knapsack problem as a tool to determine the items to be produced from furnace loading was proposed. Moreover, we proposed a genetic algorithm to explore some possible sets of alloys and to determine the production planning for a small foundry. Our method attempts to overcome the difficulties in finding good production planning presented by the method proposed in the literature. The computational experiments show that the proposed methods presented better results than the literature. Furthermore, the proposed methods do not need commercial software, which is favorable for small foundries.

1. Introduction

According to the Brazilian Foundry Association, Abifa [1], Brazil was the 7th major world producer of molten metals in 2005. At this time, the Brazilian foundry sector generated 60,000 jobs and 90% of them were in small and medium foundries (market-driven foundries). Therefore, to maintain the competitivity of this sector, it is necessary to improve the techniques to achieve high levels of efficiency and low operating costs. Production planning is one of the main factors that affects industrial productivity. In their research, Fernandes and Leite [2] discussed the importance of this planning for Brazilian foundries.

The aim of the production planning problem in market-driven foundries is to find a production plan with minimum costs of production, setup, inventory and backlogging, respecting limited resources. In the literature, there are few studies concerning the production planning problem in the foundry sector. Some of them focus on medium-sized foundries, Santos-Meza et al. [3], Araujo et al. [4], Duda [5] and Duda and Osyczka [6]. For small foundries, Silva and Morabito [7], Araujo et al. [8] and Tonaki and Toledo [9] can be cited. Good reviews for classical lot-sizing problems as Bahl et al. [10], Drexl and Kimms [11], Brahimi et al. [12] and Jans and Degraeve [13] can be cited.

In this study, we analyzed the production planning problem faced by a small foundry in the interior of Sao Paulo (Brazil), previously examined by Araujo et al. [8] and Tonaki and Toledo [9]. The lot size to be produced of each item during each period of the finite planning horizon had to be determined. This problem requires another fundamental decision, which is the choice of all the alloys that must be melted during each period. Silva and Morabito [7] present a greedy approach for the cutting and packing problem to solve this matter. Araujo et al. [8] proposed a model and a heuristic for the same problem. Based on Araujo's model, Tonaki and Toledo [9] suggested that this problem can be viewed as two sub-problems: the production planning problem of alloys and the production planning problem of items. These problems are solved in the hierarchical way. First, the alloys to be produced are determined and afterwards the items which required these alloys are defined. The authors propose Lagrangian heuristics for both of these problems. They indicate that the solution method to the problem that determines the alloy scheduling needs to be improved to create better production plans.

Our purpose is to consider the problem as suggested by Tonaki and Toledo [9], but we propose a method to evaluate different alloy production plans. We proposed a genetic algorithm as a

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solution method to determine the production plan of alloys to be melted. Once the alloys to be melted are determined, we have various lot-sizing as sub-problems. In order to solve the problem described previously, we use the knapsack problem as a tool to determine the items to be produced when the alloy is melted. This purpose focuses on presenting a solution method without commercial software and avoiding the deficiencies pointed out in the methods from Tonaki and Toledo [9]. Our solution method is considered simpler to be applied in industrial environments than the method proposed by Tonaki and Toledo [9] and computational experiments show that our purpose finds plans with lower costs.

This paper is organized as follows. In the next section, the problem is formally stated. In Section 3, we describe the solution method proposed. Test results are reported in Section 4 using instances based on real datasets. The conclusion and discussion of possible future work are presented in Section 5.

2. The production process of a small foundry

The main foundry processes melt alloys and mold metal in the required items. These processes must be scheduled at the same time. The transformation of ore and scrap metal into alloys with specified levels of carbon, silicon, zinc, etc. determines properties for items such as brittleness and corrosion resistance. The alloy in liquid state is poured into moulds to cool and produce the final items.

In this production planning problem, it is necessary to determine the lot size of the items and the required alloys to be produced during each period of the finite planning horizon. The limitation of the furnace capacity has to be respected. In the production process of the small foundry studied, only one furnace is usually in operation, so in each time period only one alloy can be processed in the furnace. An item cannot be manufactured in a given time period unless the required alloy by this item is also processed in that period. Once melted, the alloy cannot be kept for the next period. Therefore, all quantities of alloy not used in the production are scrap, generating waste, loss capacity and other damage.

All items have a demand in a planning horizon that would be produced if the furnace capacity were sufficient. In our small foundry case, delays occur because the furnace capacity is tight, and so backlogs must be represented in the objective function.

From this information, Tonaki and Toledo [9] present the problem in two steps. First, they determine the amount of alloy types to be produced. This solution involves the sum of metal quantities necessary to produce the items during each period. The second step of scheduling determines the units of each item to be produced. These are combinatorial optimization problems that can be modeled as proposed by the authors in Tonaki and Toledo [9] and it is reproduced as follows.

Indices

$k = 1, \ldots K$	type of alloys
$i = 1, \ldots N$	type of items
$t = 1, \ldots T$	time period (days, for example)
$n = 1, \ldots \eta$	sub-periods (furnace loading lasting 2 h, for example)
$F_t = 1 + \sum_{j=1}^{t-1} \eta_j$	first sub-period in period t ($F_1 = 1$)
$L_t = F_t + \eta_t - 1$	last sub-period in period t
$\eta = \sum_{t=1}^{T} \eta_t$	the total number of sub-periods over the planning horizon

Parameters

H_{kt}^-	penalty for delaying alloy k in period t
H_{kt}^+	penalty for holding alloy k in period t
s_k	setup penalty for alloy k
Cap	a single furnace loading capacity (kg)
D_{kt}	total demand of alloy k during the period t
η_t	number of sub-period in period t
ρ_i	gross weight (kg) of item i
d_{it}	number of item i ordered per period t
h_{it}^-	penalty for delaying a unit of item i in period t
h_{it}^+	penalty for holding a unit of item i in period t
$S(k)$	set of items i that use alloy k (each item uses one and only one alloy). Thus $\{1,\ldots,N\} \subset S(1) \cup \ldots \cup S(k)$, $S(h) \cap S(j) = \emptyset$, $\forall h \neq j$
Y_n^k	$Y_n^k = 1$ indicates that the furnace is setup for producing alloy k in sub-period n, otherwise, $Y_n^k = 0$

Variables

E_{kt}^-	gross weight (kg) of alloy k delayed at the end of period t
E_{kt}^+	gross weight (kg) of alloy k held at the end of period t
$\overline{Y}_t^k$	binary variable ($\overline{Y}_t^k = 1$ indicates that the furnace is setup for producing alloy k in period t, otherwise, $\overline{Y}_t^k = 0$)
A_{kt}	number of loads of alloy k that can be produced during the period t (lot size)
X_{in}	number of items i to be produced in sub-period n (lot size)
I_{it}^-	number of item i delayed at the end of period t
I_{it}^+	number of item i held at the end of period t
Z_n^k	variable related to each changeover: $Z_n^k = 1$ if there is a setup (changeover to) alloy k in sub-period n, otherwise $Z_n^k = 0$. In other words, $Z_n^k = 0$ if $Y_{n-1}^k \geq Y_n^k$ and $Z_n^k = 1$ if $Y_{n-1}^k < Y_n^k$

Production planning of the alloy problem:

$$\text{Minimize} \quad \sum_{t=1}^{T} \sum_{k=1}^{K} (H_{kt}^- E_{kt}^- + H_{kt}^+ E_{kt}^+ + s_k \overline{Y}_t^k) \tag{1}$$

subject to

$$E_{k,t-1}^+ - E_{k,t-1}^- + Cap \cdot A_{kt} - E_{kt}^+ + E_{kt}^- = D_{kt} \quad k = 1,\ldots,K; \ t = 1,\ldots,T, \tag{2}$$

$$\sum_{k=1}^{K} A_{kt} \leq \eta_t \quad t = 1,\ldots,T, \tag{3}$$

$$A_{kt} \leq \eta_t \overline{Y}_t^k \quad k = 1,\ldots,K; \ t = 1,\ldots,T, \tag{4}$$

$$\overline{Y}_t^k \in \{0,1\} \quad k = 1,\ldots,K; \ t = 1,\ldots,T, \tag{5}$$

$$A_{kt} \geq 0, integer \quad k = 1,\ldots,K; \ t = 1,\ldots,T, \tag{6}$$

$$E_{kt}^+, E_{kt}^- \geq 0 \ (E_{k0}^+ = E_{k0}^- = 0) \quad k = 1,\ldots,K; \ t = 1,\ldots,T. \tag{7}$$

The objective function (1) attempts to minimize the stock, delays and setup costs. The flow balance constraints are expressed in (2). Constraints (3) keep the production within the foundry capacity. We have the setup forcing constraint (4).

Once the alloys that should be melted in the period are determined ($\overline{Y}_t^k$ and A_{kt}), we can set up the sub-periods in which the alloys should be produced (Y_n^k). With the result that, Y_n^k are parameters for the production planning of the item problem. The production planning of the item problem can be decomposed into K independent problems, one for each alloy, showed as follows.