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Mathematical model for heat and mass transfer during convective drying of pumpkin

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ABSTRACT

Convective drying is a popular technique for drying of food materials. A plethora of empirical models have been used to fit the convective drying kinetics of food materials in general and pumpkin in particular. In literature, there exists a gap in the study of convective drying of pumpkin using first principle based model. In this article, a first principle based model for heat and mass transfer with appropriate convective boundary condition have been formulated. The model resulted in the set of partial differential equations, which have been solved using method of lines (MOL). The simulated model demonstrates good qualitative agreement with the reported literatures and experimental data. The Arrhenius parameters (activation energy and the pre exponential factor) for effective diffusivity have been estimated by fitting experimental data. Using the estimated values, the drying kinetics under various drying conditions have been simulated and found to be in good agreement with the experimental data. The different drying regimes exhibited by the pumpkin were explained using the temperature and moisture profiles predicted by the model.

1. Introduction

Pumpkin is rich in vital antioxidants, and vitamins (vitamin-A, vitamin-C, and vitamin-E) and are grown around the world at the commercial scale for its flesh and seeds. However, owing to its high moisture content (~90% w/w wet basis), pumpkin is susceptible to microbial spoilage (Roongruangsri and Bronlund, 2015). Drying helps to reduce the water activity, thereby, increasing its shelf life. Abundant work has been done on pumpkin drying by various modes including vacuum, convective, spray and freeze drying (Arévalo-Pinedo and Murr, 2007; Doymaz, 2007; Nawirska et al., 2009; Shavakhi et al., 2012; Tunde-Akintunde and Ogunlakin, 2013). Although freeze, vacuum and microwave drying produces highest quality product in terms of color, flavor, texture and composition, they suffer from high cost (Nawirska et al., 2009). For bulk economical drying of pumpkin, atmospheric convective drying appears to be the most appropriate technique mainly because of easy operation and faster drying rates.

Drying is an energy intensive process and it is imperative to find the optimum drying conditions for economical operation. Plethora of previous work on pumpkin drying uses empirical and semi-empirical models like Page model, Lewis model, Logarithmic model etc. to fit to the experimental data (Hashim et al., 2014; Doymaz, 2007; Guiné et al., 2011; Tunde-Akintunde and Ogunlakin, 2013). Empirical models give little insight to the moisture and heat transfer mechanisms and are specific to the conditions under which they are developed. They are also unable to predict the dynamic temperature and moisture profiles developed inside the food while undergoing drying operation. The knowledge of these profiles can help to estimate the certain undesirable temperature dependent physiochemical changes like color and decline in nutrient values (Jin et al., 2011). Thus, physics based model can help to optimize drying conditions for better quality control of the product.

Since drying involves simultaneous heat and mass transfer, the accuracy of the spatial and temporal profiles depends on the correctness of the effective diffusivity value. Effective diffusivity is the key variable in describing the diffusion of moisture through the food products during drying. Crank equation has been traditionally used for determination of effective diffusivity (Arévalo-Pinedo and Murr, 2006; Doymaz, 2007; Guiné et al., 2011). It is a analytical solution of the moisture diffusion equation under assumptions like isothermal drying, negligible shrinkage, negligible external resistance, constant diffusivity and uniform initial moisture content (Chen and Putranto, 2013). The main drawback of these assumptions is that the entire food piece

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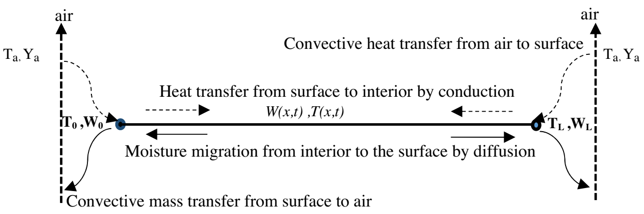

Fig. 1 – Various transfer processes during drying. *T* is the temperature, *W* is the moisture content of the pumpkin on dry basis and *Ya* is the moisture content of the drying air.

attains drying air temperature instantaneously, which is not possible under normal drying conditions. Thus, directly using Crank equation without validation of the assumptions can lead to errors in effective diffusivity's values. A more valid approach will be to express the effective diffusivity as a function of temperature inside the pumpkin cube rather than on that of drying air.

The objective of this work is twofold. First, to develop physics based model for pumpkin drying which have not been reported earlier and secondly, to estimate the effective diffusivity by fitting to the experimental data and testing its accuracy under varying operating conditions.

2. First principle model and solution methodology

Drying involves simultaneous heat and mass transfer processes as shown in Fig. 1. Heat is transferred by convection from the drying air to the pumpkin surface and then by conduction to the interior. Moisture, on the other hand moves by diffusion from the interior to the surface, undergoes a phase change and then moves to the air medium by convection. Hence, in order to simulate the drying process, it is imperative to solve the heat and moisture diffusion equations simultaneously with appropriate convective boundary conditions.

The following assumptions were made during the development of the mathematical model:

a) Phase change occurs only at the interface and not inside the pumpkin piece.
b) Shrinkage leads to reduction in the dimensions of the product being dried. This reduces the path length for the moisture to diffuse to the surface of the food product, increasing the rate of mass transfer and hence the drying rate (Shahari, 2012; Wang and Brennan, 1995). If shrinkage is not considered (the path length for diffusion remains same), the increased mass transfer rate is accounted by a higher value of effective diffusivity. Hence, ignoring shrinkage can lead to an overestimation of the effective diffusivity values. Although shrinkage is a universal phenomenon associated with drying, it was not considered in this study to keep the drying model computationally simple.
c) Initial moisture, density, composition and temperature are uniform along the spatial direction.
d) Thermal and physical properties of the moisture and the drying air are constant.

One dimensional unsteady state heat and mass transfer equations in Cartesian coordinates are given by Eqs. (1) and (2), respectively (Guiné et al., 2007; Tzempelikos et al., 2015):

$$\text{Heat diffusion}: \quad \frac{\partial \rho C_p T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial T(x,t)}{\partial x}\right) \tag{1}$$

$$\text{Moisture diffusion}: \quad \frac{\partial \rho W(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(\rho D \frac{\partial W(x,t)}{\partial x}\right) \tag{2}$$

where W (kg of moisture/kg of dry solid) is the moisture content of the pumpkin piece on dry basis, ρ is the density (mass of dry pumpkin per unit volume of pumpkin, $\mathrm{kg\,m^{-3}}$) and T is the temperature (°C). D is the effective diffusivity of the moisture through the pumpkin and is expressed as function of temperature in Arrhenius form as follows:

$$D = D_o \exp\left(\frac{-E_a}{RT(x,t)}\right) \tag{3}$$

where D_o and E_a are the pre-exponential factor ($\mathrm{m^2 s^{-1}}$) and activation energy ($\mathrm{J mol^{-1}}$), respectively.

The thermal conductivity, K ($\mathrm{W m^{-1} K^{-1}}$) variation with moisture content (for $W > 1.5$) is given as (Sweat, 1974):

$$K(W) = 0.149 + 0.493\left(\frac{W}{1+W}\right) \tag{4}$$

The dependence of specific heat, C_p ($\mathrm{J kg^{-1} K^{-1}}$), on the moisture content (W) is expressed as (Rao and Rizvi, 1995):

$$Cp(W) = 1.26 + 2.93\left(\frac{W}{1+W}\right) \tag{5}$$

The density on dry basis, ρ (kg of dry pumpkin $\mathrm{m^{-3}}$ of pumpkin) is related to the density of fresh pumpkin, ρ_o (kg of material $\mathrm{m^{-3}}$ of material) as:

$$\rho = \rho_o\left(\frac{1}{1+W}\right) \tag{6}$$

Convective boundary conditions for the heat and mass transfer (Guiné et al., 2007) are given by Eqs. (7) and (8), respectively. Due to the symmetry of the system, these equations can be applied to both the extreme nodes i.e. $x = 0$ and $x = L$:

Heat transfer boundary condition :

$$-h(T_a - T_s) = -K\left.\frac{\partial T}{\partial x}\right|_{0,L} + H_v \rho D \left.\frac{\partial W}{\partial x}\right|_{0,L} \tag{7}$$

Mass transfer boundary condition :

$$\rho D \left.\frac{\partial W}{\partial x}\right|_{0,L} = k_g(Y_s - Y_a) \tag{8}$$

The Eq. (7) represents the boundary condition for heat transfer. The left hand side of the Eq. (7) is the total heat transfer due to convection at the boundary, where h is the heat transfer coefficient and T_a and T_s are the temperature of the drying air and the pumpkin surface, respectively. The first term on the right hand side represents the heat flux at the surface of the pumpkin, whereas the second term accounts for the heat required to vaporize the moisture present at the surface, where H_v is the latent heat of vaporization ($\mathrm{J kg^{-1}}$). The mass transfer boundary condition (Eq. (8)) provides the continuity in the moisture transfer such that the moisture migrating to the surface of the pumpkin through diffusion (left hand side) is transferred to the drying air by convection (right hand side), where k_g is the mass transfer coefficient ($\mathrm{m s^{-1}}$). Y_a and Y_s ($\mathrm{kg\,m^{-3}}$) are the absolute humidity of the drying air and the moisture concentration at the surface, respectively.