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Sequencing surgical cases in a day-care environment: An exact branch-and-price approach

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ABSTRACT

In this paper we investigate how to sequence surgical cases in a day-care facility. We specify a multi-criteria objective function in which we minimize the peak use of recovery beds, the occurrence of recovery overtime and the violation of various patient and surgeon preferences. The limited availability of resources and the occurrence of medical precautions, such as an additional cleaning of the operating room after the surgery of an infected patient, are taken into account. We apply column generation to solve this combinatorial optimization problem and propose a dynamic programming algorithm to solve the pricing problem. The computational efficiency of this dynamic programming approach is validated through comparison with a mixed integer linear programming approach. In order to obtain integer variables, we embed the column generation loop in an enumerative branch-and-price framework. We elaborate on various branching strategies and branching schemes and examine their impact on the solution quality. The test instances for the computational experiments are generated using real-life data of the surgical day-care center at the academic hospital UZ Leuven Campus Gasthuisberg (Belgium).

1. Introduction

In a hospital, the operating theater is a major cost driver that unites many stakeholders, like surgeons, managers, trustees or nurses, who may have conflicting preferences and priorities [1]. Due to this inherent complexity, it is hard to construct an effective and efficient surgery schedule. The field of operations research and operations management may assist in the development of such a schedule and consequently contribute to the performance of a hospital as a whole [2].

The surgery scheduling process of elective cases can be classified by four stages or planning levels. In a first stage, one determines how much operating room time is assigned to the different surgeons or surgical groups. This stage is often referred to as case mix planning and is situated on a strategic level [3]. The second stage, which is tactically oriented, often concerns the development of a master surgery schedule. This schedule defines the number and type of operating rooms available, the hours that rooms will be open, and the surgeons or surgical groups to whom the operating room time is assigned. In the literature, many approaches for constructing master surgery schedules are cyclic [4–6]. Next to the development of a master surgery schedule, other tactical planning problems are considered in the literature [7,8]. In the third stage, individual patients or cases are scheduled on a daily base. Methodologies for scheduling individual surgical cases are often based on a two-step procedure. The first step describes the assignment of patients to days. In the second step, the patient population for a specific day is sequenced. Solution procedures that distinguish between these two phases can, for instance, be found in [9–13]. Finally, there is a fourth stage in which the enrolment of the surgery schedule is monitored online. When uncertainties materialize and the surgery schedule is substantially disrupted, rescheduling may be necessary.

This research focuses on the sequencing step of the third stage and elaborates on the surgical case sequencing problem (SCSP) that was addressed in [14]. However, now we apply a branch-and-price technique to this NP-hard optimization problem instead of straightforward mixed integer linear programming (MILP) approaches. In contrast to the exact MILP procedures [14], the branch-and-price procedures of this paper are successful in finding at least one feasible solution within the limited time frame and result in both a smaller average solution gap and a smaller standard deviation of this solution gap. Moreover, the branch-and-price procedures do not perform worse than the iterated MILP procedure [14], which is a heuristic (see Section 5). Note that the solution gap points at the potential

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progress in the objective function that possibly can be achieved by modifying the current sequences. Although the algorithms are deterministic in nature, this should not present a major drawback as we are examining a day-care environment in which difficult, rare and highly uncertain surgeries are typically not performed and procedures are more or less standardized.

In order to augment the applicability and relevance of the developed algorithms, we maintain a steady cooperation with the surgical day-care center of the academic hospital UZ Leuven Campus Gasthuisberg in Leuven (Belgium). This medical facility has already been the subject of research in a case study of Beliën et al. [15] and yearly accounts for about 15 000 hours of total net operating time and 13 000 ambulatory surgeries, i.e. surgeries of patients who are admitted and discharged on the same working day. Using a questionnaire in 2004, the International Association for Ambulatory Surgery revealed a rising trend in ambulatory surgery amongst its member countries because of the progress in surgical expertise and the introduction of new anaesthetic medications [16]. Hospitals furthermore strive to reduce the length of stay of patients, which also contributes to the increased share of ambulatory surgery. In Belgium, the share of ambulatory surgery already equals 30% of the total surgical activity.

The current sequencing approach at the day-care center results from negotiations between the surgeons and the head nurse of the operating theater. While surgeons in general limit their scope to their individual preferences, the head nurse focuses on the quality of the schedule as a whole. Although this methodology is common practice since the opening of the day-care center in 2002, it has some major disadvantages. Changes made by the head nurse, for example, are often perceived as unfair. Moreover, these changes are induced by rules of thumb that do not cover complex interactions, such as the demand for recovery beds. The process is furthermore very time-consuming due to the lack of an efficient software support system. The algorithmic solution developed in this paper will assist the head nurse in generating fair and improved surgery schedules which surpass the level of detail of the hand-made schedules by far.

The remainder of this paper is structured as follows. Section 2 discusses the SCSP and captures its multiple objectives and constraints in a pattern-based mathematical formulation. Section 3 decomposes the SCSP and describes a column generation approach. Since column generation cannot guarantee variables to be integer, we extend this methodology to a broad branch-and-price framework in Section 4. We propose multiple branching schemes and combine them with the column generation algorithm. In Section 5, a detailed computational experiment is conducted using data from the surgical day-care center. Finally, in Section 6 we formulate conclusions and mention ideas for future research. We refer to Appendix A for a complete overview of the symbols used in this paper.

2. Problem statement

The SCSP maximally comprises six objectives ($|J| \leqslant 6$). First, we want the surgeries of children (age $\leqslant 5$ years) to be performed as early as possible during the day since it is hard for them to remain sober. Second, we also want prioritized patients to be scheduled as early as possible in order to protect them from delays or possible cancelations. Third, we incorporate the travel distance between the patient's residence and the day-care center. We want to schedule patients with a substantial travel distance after a certain reference period. When such patients have more time to manage the trip, there is an increase in patient satisfaction while there is less chance to arrive late at the hospital due to traffic uncertainty. Fourth, we want to minimize the stay in recovery after the closure of the day-care center, since this results in unplanned (and hence costly) hospitalizations or overtime for the nursing personnel. Finally, we are interested in minimizing the peak number of beds used in post-anesthesia care unit (PACU) 1 and PACU 2 in order to level the bed occupancy and hence level the workload of the nursing personnel. We distinguish between the two recovery units because of the degree of monitoring. After surgery, patients are transferred to PACU 1 in order to get through the critical awakening phase. When the patient is conscious and the awakening process in PACU 1 tends to be normal, the patient is moved to PACU 2 where the patient stays until the surgeon gives permission to leave the day-care hospital.

We combine the objectives as represented in Expression (1). In this function, α_1 (α_2) equals the sum of the surgery starting times of children (prioritized patients), α_3 equals the number of travel patients that is scheduled before the reference period, α_4 equals the total amount of recovery overtime (expressed in periods) and α_5 (α_6) equals the peak number of beds used in PACU 1 (PACU 2). Since several objectives are expressed in different units, a trade-off has to be defined. This decision, however, is very subjective and hence difficult to argument. Therefore, we propose a type of normalized objective function that originates from the field of multiple criteria decision making (e.g. [17]). Since the patient population is known, we should be able to calculate for each single objective j its best value ($bestvalue_j$) and its worst value ($worstvalue_j$). These extreme values are consecutively used as indicated in Expression (1) to generate a relative measure of quality that is easy to interpret. When $bestvalue_j$ equals $worstvalue_j$, we do not take the optimization of objective j into account. This implies that the value of objective j is optimal for every feasible schedule and that $0 \leqslant |J| \leqslant 6$, depending on the problem characteristics. The stabilizer or normalization ensures that all objectives $j \in J$ will be gradually optimized to the same extent and will somehow be comparable to each other. It is unlikely, however, that the objectives are of equal importance to the human planner. Thus, we also incorporate a differentiator by assigning a weight w_j to objective j. Note that the weights only indicate the preferences of the scheduler. When the sum of the weights equals 1, the multiple objective function has a value that is in the range [0,1].

$$\sum_{j \in J} w_j \cdot \left(\frac{\alpha_j - bestvalue_j}{worstvalue_j - bestvalue_j} \right) \tag{1}$$

The extreme values for the objectives are obtained by solving the SCSP using the preprocessed MILP model that is addressed in [14]. Now, though, we have to introduce variants on the objective function, both in sense and in formulation. For each objective j the model is solved twice, namely minimizing α_j to obtain $bestvalue_j$ and maximizing α_j to obtain $worstvalue_j$. It should be noted that maximizing α_5 or α_6, respectively, results in the total capacity provided in PACU 1 (= PACU1cap) or PACU 2 (= PACU2cap), while this is not necessarily a true upper bound. Therefore we solve for each objective $j \in \{5,6\}$ a set of $|P|$ subproblems, i.e. for each period $p \in P$, which is the set of 5-minute periods that constitute the working day, we solve a problem in which we maximize the number of beds used in that specific period. The worst value for the objectives $j \in \{5,6\}$ is consecutively set equal to the highest solution value obtained over the respective $|P|$ subproblems. In Section 5 we evaluate the computational efficiency of this procedure.

Let z_{st} denote a binary decision variable that equals 1 if column t is chosen for surgeon s. A column can be seen as a sequenced group in which all surgeries for one specific surgeon are represented. Let a^e_{pst} represent the number of instruments of type e in use at period p when column t is chosen for surgeon s and let a^j_{pst} ($j \in J : j \geqslant 5$) indicate how many beds of PACU 1 (j=5) or PACU 2 (j=6) are needed in period p when column t is chosen for surgeon s. Next to these resource parameters, we also have a cost parameter c^j_{st} that indicates how much α_j will increase when column t is chosen for surgeon s ($j \in J : j \leqslant 4$). Furthermore, let $PACU1cap$ ($PACU2cap$) represent the number of beds available in PACU 1 (PACU 2) and let cap_e denote the