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A linear programming method for generating the most favorable weights from a pairwise comparison matrix

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Abstract

This paper proposes a linear programming method for generating the most favorable weights (LP-GFW) from pairwise comparison matrices, which incorporates the variable weight concept of data envelopment analysis (DEA) into the priority scheme of the analytic hierarchy process (AHP) to generate the most favorable weights for the underlying criteria and alternatives on the basis of a crisp pairwise comparison matrix. The proposed LP-GFW method can generate precise weights for perfectly consistent pairwise comparison matrices and approximate weights for inconsistent pairwise comparison matrices, which are not too far from Saaty's principal right eigenvector weights. The issue of aggregation of local most favorable weights and rank preservation methods is also discussed. Four numerical examples are examined using the LP-GFW method to illustrate its potential applications and significant advantages over some existing priority methods.

Keywords: Analytic hierarchy process; Multicriteria decision analysis; Data envelopment analysis; Most favorable weights; Rank preservation

1. Introduction

In the analytic hierarchy process (AHP) literature the derivation of priorities of criteria or alternatives from their pairwise comparisons has been investigated extensively. Apart from Saaty's eigenvector method (EM) [1], which is the most widely used priority method, Chu et al. [2] proposed a weighted least-squares method (WLSM). Crawford [3] proposed a logarithmic least-squares method (LLSM). Saaty and Vargas [4] presented a least-squares method (LSM). Cogger and Yu [5] suggested a gradient eigenweight method (GEM) and a least distance method (LDM). Islei and Lockett [6] developed a geometric least-squares method (GLSM). Bryson [7] put forward a goal programming method (GPM). Bryson and Joseph [8] also brought forward a logarithmic goal programming approach (LGPA). Mikhailov [9,10] proposed a fuzzy programming method (FPM). Lipovetsky and Conklin [11] presented a robust estimation method (REM). Beynon [12–14] developed a DS/AHP method, which integrates the Dempster–Shafer theory of evidence [15] into the AHP. Gass and Rapcsák [16] offered a singular value decomposition (SVD) approach. Laininen and Hämäläinen [17] analyzed AHP-matrices by robust regression. Stam and Silva [18] discussed a multiplicative priority rating method, which is a variant of AHP. Sugihara et al. [19] suggested an interval priority method called

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possibilistic AHP for crisp data (PAHPC). Chandran et al. [20] presented an approach based on linear programming (LP) for estimating weights. Srdjevic [21] suggested combining different prioritization methods. Wang et al. [22] proposed a correlation coefficient maximization approach (CCMA), which estimates priorities through maximization of correlation coefficient. A comprehensive survey of priority methods is available in Wang [23], where 21 priority methods were summarized and compared. Vaidya and Kumar [24] provided an overview of AHP applications.

Recently, Ramanathan [25] developed an approach combining data envelopment analysis (DEA) and AHP, which they called DEAHP for weight derivation and aggregation. Although the combination of DEA and AHP was not new (see, for example, [26–28]), DEA had not been used to derive priorities from a pairwise comparison matrix. DEAHP employs DEA to generate the local weights of alternatives from pairwise comparison matrices and aggregate them into final weights. It is claimed that rank reversal does not occur in DEAHP when an irrelevant alternative is added or removed. But so far as we can determine, DEAHP has some significant drawbacks. First of all, it fails to make use of all the information on pairwise comparisons when deriving individual local weights. Only the judgment elements in one or two columns of a pairwise comparison matrix are used each at a time. Second, the local weights generated by DEAHP may be quite irrational even wrong for a highly inconsistent pairwise comparison matrix. Finally, DEAHP still suffers from the rank reversal problem if a DEA efficient alternative is added or removed. These drawbacks make it necessary to develop a new methodology that is able to overcome the mentioned drawbacks.

In this paper we propose an LP method that integrates the variable weight concept of DEA into AHP to generate the most favorable weights for criteria or alternatives on the basis of a matrix of pairwise comparisons. We call this method a linear programming method for generating the most favorable weights, or LP-GFW for short. LP-GFW method differs from the LP-based approach presented by Chandran et al. [20] in that it uses variable weights for each criterion or alternative and consists of n LP models, while Chandran et al.'s uses fixed weights and is composed of a two-stage goal programming model. The proposed LP-GFW method is not hampered by the limitations of DEAHP and makes a new contribution to the priority theory of AHP and its applications.

The paper is organized as follows. Section 2 briefly reviews DEAHP. Section 3 lays the theoretical foundations of LP-GFW. Section 4 addresses the issue of aggregation of local most favorable weights and presents a rank preservation method. Four numerical examples are provided and analyzed in Section 5. The paper is concluded in Section 6.

2. DEAHP: combining DEA and AHP

Let

$$A = (a_{ij})_{n\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{1}$$

be a pairwise comparison matrix with $a_{ii} = 1$ and $a_{ji} = 1/a_{ij}$ for $j \neq i$ and $W = (w_1, \ldots, w_n)^{\mathrm{T}}$ be its weight vector. DEAHP views each row of the matrix A as a decision making entity, which is referred to as decision making unit (DMU) in DEA, and each column as an output. A dummy input that has a value of 1 is assumed for all the DMUs. Each DMU has in fact n outputs and one dummy constant input, based on which the following input-oriented CCR model is built to estimate the local weights of the comparison matrix A:

$$\text{Max} \quad w_{i_0} = \sum_{j=1}^{n} a_{i_0 j} v_j$$

$$\text{s.t.} \quad \begin{cases} u_1 = 1, \\ \sum_{j=1}^{n} a_{ij} v_j - u_1 \leqslant 0, & i = 1, \ldots, n, \\ u_1, v_j \geqslant 0, & j = 1, \ldots, n, \end{cases} \tag{2}$$

where $i_0 \in \{1, \ldots, n\}$ represents the DMU under evaluation, that is DMU_0. The optimum objective function value of the above model, $w_{i_0}^*$, represents the DEA efficiency of DMU_0 and is used as its local weight. The LP model (2) is solved for all the DMUs to obtain the local weight vector $W^* = (w_1^*, \ldots, w_n^*)^{\mathrm{T}}$ of the comparison matrix A. It has