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Maximising expectation of the number of transplants in kidney exchange programmes

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ABSTRACT

This paper addresses the problem of maximising the expected number of transplants in kidney exchange programmes. New schemes for matching rearrangement in case of failure are presented, along with a new tree search algorithm used for the computation of optimal expected values. Extensive computational experiments demonstrate the effectiveness of the algorithm and reveal a clear superiority of a newly proposed scheme, subset-recourse, as compared to previously known approaches.

1. Introduction

Kidney exchange programmes have been organised in many countries as an alternative mode of transplant for patients with kidney failure that have a donor willing to provide a kidney, but the pair is not physiologically compatible [9,4,24,25,1]. These programmes are based on the concept of "exchange" between patient–donor pairs: donors are allowed to provide a kidney to patients in the other pairs, if compatibility exists, so that patients in all pairs involved in the exchange benefit.

Fig. 1 (left) illustrates the simplest case with only two pairs, (P_1, D_1) and (P_2, D_2). Donor D_1 of the first pair is allowed to give a kidney to patient P_2 of the second pair, and patient P_1 may get a kidney from donor D_2. In the right-hand side of the figure an exchange between three incompatible pairs is possible: patient P_1 may receive the kidney from donor D_3, patient P_2 from donor D_1, and patient P_3 from donor D_2; in that case all patients are served. Notice that these graphs consider only preliminary compatibilities that will have to be reassessed prior to actual transplant through additional medical exams.

For logistical reasons, and also to reduce the number of affected patients when last-minute donor resignation occurs or new incompatibilities are detected, a limit k is imposed on the length of cycles.

The optimisation problem underlying a basic kidney exchange commonly considers the maximisation of the number of transplants [9,25]. But other criteria have been proposed to be maximised, e.g., the number of effective 2-cycles [18], the number of transplants with identical blood type [14], the overall score or utility assigned to each transplant [17,18], and the expected number of transplants [20]. For k limited and greater than 2, the problem of maximising the number of transplants is known to be NP-hard [4,1,15]. Integer programming (IP) is a natural framework for modelling this optimisation problem. A summary of IP models for the kidney exchange problem (KEP) has been presented in [8,16], where several formulations are analysed and compared in terms of tightness and experimental performance.

The most common formulations consider that there is no uncertainty associated with the data, which is not the case in practice. In fact, one of the main problems in the implementation of the solution of a KEP instance arises from data unreliability. Last-minute testing of donor and patient can elicit new incompatibilities (so-called, positive cross-matching) that were not detected before, causing some donations in a cycle to be cancelled; patients or donors may become unavailable, e.g., due to illness or to backing out. Data uncertainty is addressed by some authors by associating probabilities of failure to vertices (pairs) and arcs (compatibilities) and by considering the expected size of cycle, rather than the actual one.

A model considering a discounted utility of cycles was proposed in [11]. It takes into account a probability of failure, but rearrangement of vertices in case of failure is not considered (we will call this model "*no-recourse expectation*" in Section 3.1). A

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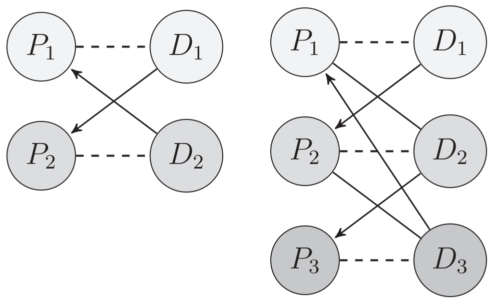

Fig. 1. An exchange between two (left) and three (right) incompatible pairs. Solid lines represent preliminary assessment compatibility and arrows define a possible exchange.

model for maximising the expected utility when arcs are subject to failure is proposed in [17] and [7]. A simulation system, where patient–donor pairs are generated and assigned to a cycle in a dynamic version of the KEP, is also proposed there. An approach for maximising expectation was studied in [20], considering both vertex and arc failure. A straightforward scheme was used for the computation of expectation, which turned out to be very limited; only experiments with k equal to 3 have been performed.

The main contributions of this paper are the following. We propose a new tree search algorithm for calculating the expected number of transplants in a KEP which allows instances of the KEP for larger values of k to be handled, as compared to the approach used in [20]. We also propose a new scheme for the rearrangement of vertices on cycles with failure — thus, with a different value for the expectation — where the recovery of broken cycles may be performed with rearrangements within a wider subset of vertices, which we believe is implementable in practice. A computational experiment was carried out to compare the different rearrangement schemes, as well as to test the algorithm used for calculating expectations. The results obtained show that by using the new rearrangement scheme a meaningful increase on the number of transplants is possible for most of the instances.

The remainder of this paper is organised as follows. A formal problem statement and IP formulation is provided in Section 2. In Section 3, three expectation schemes are presented, together with the algorithms for their computation. Results of computational experiments are presented in Section 4, which is followed by the conclusions.

2. Problem statement and IP formulation

Graph theory provides a natural framework for representing kidney exchange models. Given a directed graph $G=(V,A)$, the set of vertices V is the set of incompatible donor-patient pairs. Two vertices i and j are connected by arc $(i,j)\in A$ if donor of pair i is compatible with patient of pair j. To each arc (i,j) is associated a weight w_{ij}. If the objective is to maximise the total number of transplants, then $w_{ij}=1,\ \forall (i,j)\in A$.

The kidney exchange problem (KEP) can be defined as follows: *Find a packing of vertex-disjoint cycles with length at most k having maximum weight.*

There are several known integer programming (IP) formulations for the KEP [8]. The work presented in this paper is based on one of the computationally most effective formulations — the cycle formulation [23] — which can be described as follows. Let C be the set of all cycles in G with length at most k. We represent a cycle as an ordered set of arcs. Define variable $x_c=1$ if cycle $c\in C$ is selected for the exchange, $x_c=0$ otherwise. Denote by $V(c)\subseteq V$ the set of vertices which belong to cycle c and by w_c the weight of cycle: $w_c=\sum_{(i,j)\in c} w_{ij}$. The cycle formulation can be written as follows.

Problem $\mathcal{C}(\mathbf{k})$: (1)

$$\text{maximise} \quad \sum_{c\in C} w_c x_c \tag{2}$$

$$\text{subject to}: \quad \sum_{c:i\in V(c)} x_c \le 1 \quad \forall i\in V, \tag{3}$$

$$x_c\in\{0,1\} \quad \forall c\in C. \tag{4}$$

The objective function (2) maximises the weighted sum of the exchange. Constraints (3) ensure that each vertex is in at most one of the selected cycles (so that the corresponding patient/donor will at most receive/donate one kidney). The difficulty of this formulation is induced by the exponential number of variables in terms of k, related to the exponential number of cycles of length at most k (in the general case).

State-of-the-art kidney exchange programmes include altruistic donors, i.e., donors that are not associated with any patient, but are willing to donate a kidney to someone in need. In a kidney exchange programme, an altruistic donor initiates a chain, not a cycle: she/he gives a kidney to a patient and the recipient's donor is “dominoed” to add another incompatible pair to the chain and so on. The last donor in the chain normally gives a kidney to the next compatible patient on the deceased donors waiting list [19,13,22]. European programmes consider bounded chains with length at most k' (k' can be different from k) [18,14]. A discussion on how to extend the cycle formulation to include altruistic donors is provided in [8]. Alternatively, we may have a *non-simultaneous extended altruistic donor chain* [10,21,2,12] (also known as never-ending-altruistic-donor chain), where a kidney from the last donor in a chain (called a “bridge” donor) is not assigned to a patient in the deceased donors list; instead, the bridge acts as an altruistic donor for future matches. The cascading donor chain may continue indefinitely and the length of the chain is unbounded, unless a donor whose related recipient has already been transplanted drops out of the programme. Note that in this case simultaneity of operations is no longer a requirement. This possibility was adopted by some programmes in the USA. The inclusion of altruistic donor chains is not considered explicitly in this paper.

3. Unreliability and recourse policies

According to [11], in a particular kidney exchange programme running in the USA only 7% of the matches resulted in transplants, while 93% failed. For taking into account this observation – common in these programmes – alternative objectives for the KEP should be considered. Instead of using the cycle's length as a weight of the cycle (the most common approach [9,25]), one may consider using the *expectation* on the number of transplants that the cycle will lead to. This expectation can be computed if the probability of failure of each of the cycle's vertices and arcs is given. In this paper we will refer to the probability of failure of vertex $i\in V$ in the graph as p_i, and to the probability of failure of arc $(i,j)\in A$ as p_{ij}. Throughout the paper it is assumed that the failure events are statistically independent, and that arcs have unit weights, $w_{ij}=1\ \ \forall (i,j)\in A$. We will consider three recourse schemes for the rearrangement of exchange cycles. We revisit the no-recourse and internal-recourse policies and propose a subset-recourse policy. We also propose algorithms for computing the corresponding expectations.