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Scheduling a deteriorating maintenance activity and due-window assignment

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ABSTRACT

Several papers published during the last decade dealt with scheduling a maintenance activity and considered a new setting, where the maintenance duration is assumed to be *deteriorating*, i.e., it requires more time or effort if it is delayed. We study a deteriorating maintenance in the context of due-window assignment, where a time interval is determined such that jobs completed within this interval are "on-time", whereas early and tardy jobs are penalized. Thus, our paper extends known models by considering *simultaneously* a deteriorating maintenance and due-window. Two deterioration types are considered: *time-dependent* (where the maintenance time increases as a function of its starting time), and *position-dependent* (where it is a function of its position in the sequence). The classical assumption of position-independent processing times was considered first, and then the model is extended to *general position-dependent* processing times. We prove several properties of the optimal timing of the due-window and of the maintenance. Consequently, we show that all the problems studied here are solved in $O(n^4)$, where n is the number of jobs.

1. Introduction

In a recent paper, Yang [21] studied single machine scheduling problems with the option to assign maintenance activities during the production process. The underlying assumption in these models is that, as in many real-life settings, the maintenance activities are *deteriorating*, i.e. they require more time or effort if they are delayed. This new line of research considering deteriorating maintenance activities was introduced first by Kubzin and Strusevich [7]. Following their idea, several other researchers considered various similar settings: Mosheiov and Sidney [14] studied several objective functions such as maximum lateness and number of tardy jobs, Yang [20] studied single machine scheduling problems with both start-time dependent learning and position dependent aging effects, Yang and Yang [23] focused on minimizing the total completion time on a single-machine scheduling with aging/deteriorating effects, Wang et al. [18] investigated parallel machine settings, Cheng et al. [4] and Yang et al. [24] considered a common due window assignment with linear time-dependent deteriorating jobs, and Cheng et al. [3] and Yang et al. [22] focused on deteriorating maintenance on unrelated parallel machines.

In this paper we study a problem of scheduling a maintenance activity on a single machine, with a *due-window assignment*. The basic due-window assignment problem has been introduced and solved by Liman et al. [10], who considered the following setting: n jobs need to be processed on a single machine around a common due-window; jobs completed within the due-window are "on-time" jobs, whereas early and tardy jobs are penalized. Four cost components were included in the objective function assumed by Liman et al. [10]: earliness, tardiness, due-window starting time and due-window size. They introduced an $O(n \log n)$ solution, consisting of the job sequence and the determination of the due-window size and location.

Mosheiov and Sarig [12] extended the problem to a setting allowing an option of performing a maintenance activity. On the one hand, this activity requires a fixed (given) time interval during which the machine is turned off, and the following jobs are delayed. On the other hand, the maintenance activity is assumed to be a *rate modifying* activity, i.e. the machine efficiency is improved as a result of the maintenance, and the processing times of the following jobs are shortened. Lee and Leon [8] introduced the first model with a rate-modifying activity. They were followed by Lee and Lin [9], Kubzin and Strusevich [7], Gordon and Tarasevich [5], Zhao et al. [26], Lodree and Geiger [11], Jin and Cheng [6], and Zhao and Tang [25], Wang and Wang [19], among others. The problems solved in these papers are clearly harder, as they require in addition to the

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traditional job scheduling and due-window assignment decisions, a decision regarding the optimal timing of the maintenance activity.

In this paper we extend the above models by combining a due-window assignment and the option of scheduling a *deteriorating* maintenance activity. In this setting the maintenance time increases if it is delayed. As written in Mosheiov and Sidney [14], "the longer the maintenance activity is delayed, the worse the system conditions become, so that the maintenance requires more effort and time". We assume two deterioration types: *time-dependent* deterioration and *position-dependent* deterioration. First, as in Kubzin and Strusevich [7], we assume *linear* deterioration, i.e. the maintenance time increases linearly with its starting time. Then, we assume that the deterioration time increases (in the most general form) as a function of its position. Consider the following typical example of a car maintenance: the driver is required to go through a maintenance procedure either after a certain amount of time (reflecting time dependent deterioration), or a specific distance traveled (position-dependent deterioration). We show that in both cases the extension does not increase the complexity of the problem (compared to the associated due-date assignment model). We further extend these models to allow general *position-dependent* job processing times. These extensions do not increase the total complexity as well, i.e. the problems are solved in $O(n^4)$ time, where n is the number of jobs. Several recent papers appear to be particularly relevant. First, we note that Cheng et al. [4] and Yang et al. [24] studied due-window assignment with a deteriorating maintenance as well. However, in their models: (i) the job processing times are deteriorating with *job-independent* deterioration parameters, and (ii) after performing the maintenance, the machine reverts to its initial condition. Their problems are shown to be solved in $O(n^2 \log n)$ time. The maintenance considered in our model is, as mentioned, a *rate modifying* activity, and the effect on the job processing times is *job-dependent*. Rustogi and Strusevich [15] studied a more general model, in which the scheduler may perform more than a single maintenance activity. Unlike previous papers dealing with scheduling multi-maintenance activities, Rustogi and Strusevich [15] considered a very realistic assumption that the maintenance does not necessarily fully restore the machine to its original state. We note that this model (i) does not consider a due-window (the objective function is minimum makespan, whereas our objective is minimum earliness, tardiness and due-window costs), and (ii) assumes only *deteriorating* job processing times, where we allow *general* position dependent processing times (and no monotonicity is enforced). Rustogi and Strusevich [16] studied a very general scheduling model, where the contribution of a job to the objective function is given by a product of its processing time and a certain positional weight. It is well known that many scheduling problems fall into this category and, depending on the input matrix, can either be reduced to a linear assignment problem, or even be solved by simple matching. The input matrix considered in their paper is assumed to be determined by a product of two arrays (i.e., $C_{ij} = \alpha_i \beta_j$ for the given arrays $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $(\beta_1, \beta_2, \ldots, \beta_n)$). It should be noted that the input matrix assumed in our paper (for the case of position-dependent processing times) is *not restricted* in any way. Finally, Rustogi and Strusevich [17] studied another extension of a model dealing with a single machine and several (potential) maintenance activities, in which the actual processing times of the jobs are subject to a combination of positional and time-dependent effects. However, unlike the model studied here, these effects in Rustogi and Strusevich [17] are *job-independent*. Thus, in our knowledge, the problem combining (i) a due-window assignment, (ii) scheduling a deteriorating maintenance, and (iii) general position-dependent job processing times, is studied here for the first time.

In Section 2 we provide the notation and the formulation of the problem. Section 3 summarizes a list of properties of an optimal schedule. Sections 4 and 5 present the solutions for the cases of time-dependent deterioration and of position-dependent deterioration, respectively.

2. Formulation

We study an n-job, single-machine scheduling problem. An option of scheduling a maintenance activity (rate modifying) may be considered by the scheduler. First, as in Lee and Leon [8], we assume that the length of the maintenance is a function of its starting time. If it starts at time t, then its length is $T_{MA}(t) = t_0 + \omega t$, where $t_0 > 0$ is its basic maintenance time (if performed at time zero), and $\omega > 0$ is a non-negative deterioration factor. The case of a linear decreasing maintenance time (learning) is also considered: $T_{MA}(t) = t_0 - \omega t$. Then, we assume that the maintenance is a function of its position: if it is scheduled before the job in position r, it requires $T_{MA}(r) = \tau_r$ time, where τ_r is a position-dependent constant, and $\tau_r \le \tau_{r+1}$, $r = 1, \ldots, n-1$. During the maintenance no production is performed. In the simplest model studied here, the processing times of the jobs scheduled prior to the maintenance are assumed to be position-independent, and similarly, the processing times of the jobs scheduled after the maintenance are assumed to be position-independent. Specifically, the processing time of job j is P_j if the job is processed prior to the maintenance activity, and $\theta_j P_j$ $(0 < \theta_j \le 1)$ if it is scheduled after it, $j = 1, \ldots, n$. θ_j is the *modifying rate* of job j. We also use the notation P_r and θ_r to denote the processing time of the job in the r-th position, and the modifying rate of the job in the r-th position, $r = 1, \ldots, n$, respectively. This basic model is extended to allow position-dependent processing times. In this case P_{jr} denotes the processing time of job j if assigned to position r, $j, r = 1, \ldots, n$.

For a given job sequence, the completion time of the job in position r is denoted by C_r, $r = 1, \ldots, n$. All the jobs share a common due-window: $[d_1, d_2]$, such that $d_1 \le d_2$. The earliness of the job in the r-th position is given by $E_r = \max\{0, d_1 - C_r\}$ and the tardiness of the job in the r-th position is given by $T_r = \max\{0, C_r - d_2\}$, $r = 1, \ldots, n$. The objective function consists of four cost components: α is the earliness unit cost, β is the tardiness unit cost, γ denotes the unit cost of (delaying) the due-window starting time, and δ is the unit cost of (increasing) the due-window size. Let $D = d_2 - d_1$ denote the due-window size. Then the objective function is (see [10]):

$$Z = \sum_{r=1}^{n} (\alpha E_r + \beta T_r + \gamma d_1 + \delta D).$$

3. Basic properties: due-window and maintenance location

We quote a number of properties of an optimal schedule proved in Mosheiov and Sarig [12] for the case of a *fixed* maintenance time. These properties are easily shown to be valid for our more general setting of deteriorating maintenance.

Property 1. An optimal schedule starts at time zero and contains no idle time between consecutive jobs.

Property 2. An optimal schedule exists in which the maintenance activity is performed prior to the starting time of one of the jobs, or not performed at all.

Define:

Condition (i): $\gamma > \delta$.
Condition (ii): $\beta < \min\{\gamma, \delta\}$.

Property 3.1. If condition (i) holds, an optimal schedule exists in which the due-window starts at time zero.