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A column generation approach to extend lifetime in wireless sensor networks with coverage and connectivity constraints

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ABSTRACT

This paper addresses the maximum network lifetime problem in wireless sensor networks with connectivity and coverage constraints. In this problem, the purpose is to schedule the activity of a set of wireless sensors, keeping them connected while network lifetime is maximized. Two cases are considered. First, the full coverage of the targets is required, and second only a fraction of the targets has to be covered at any instant of time. An exact approach based on column generation and boosted by GRASP and VNS is proposed to address both of these problems. Finally, a multiphase framework combining these two approaches is built by sequentially using these two heuristics at each iteration of the column generation algorithm. The results show that our proposals are able to tackle the problem efficiently and that combining the two heuristic approaches improves the results significantly.

1. Introduction

Wireless sensors are small devices with low energy consumption rates that are typically deployed to monitor some interesting phenomena, *e.g.* surveillance, military applications, environmental monitoring, etc. [8,30]. In wireless sensor networks (WSNs) deployed to monitor targets, these devices work collaboratively or individually to collect information from the field and to deliver or spread the collected data to a remote base station through a multihop path of active sensors.

Energy consumption is a major concern for the implementation and deployment of WSN [6,27]. Furthermore, there exists an extended range of applications in which the replacement of sensors or the renewal of batteries is not feasible, like in hostile or contaminated environments. This fact stresses the necessity of designing efficiently schemes for the simultaneous use of sensors energy.

In order to keep the network operating as long as possible, a common strategy is to deploy more sensors than actually needed. Then, network lifetime can be extended by activating sequentially subsets of sensors able to meet the network requirements. The sensing range R_s is defined as the maximum distance a sensor can cover a target. Two sensors are considered connected if the distance between them is less than the communication range R_c (in practice, $R_s \leq R_c$). Only the sensors from an active set are available for monitoring targets and transmitting the collected data. So, the optimal use of network energy can be obtained by identifying and creating schedules for the use of the sensors in the network.

In some applications, the complete collection of information originated in the targets is not a critical requirement. Thus, a threshold can be defined as the minimum level of coverage provided by the network, *i.e.* the fraction α of targets that has to be covered at any instant of time. This characteristic provides the network with a bit of flexibility which, in addition, allows us to increase its lifetime by neglecting some of the targets that are poorly covered and become a bottleneck limiting the network lifetime [14].

In order to optimize the usage of the energy in WSN, researchers have addressed the maximum network lifetime problem (MLP) [6,9,19,27]. This problem consists in maximizing the lifetime of a WSN while guaranteeing the coverage of a discrete set of targets. Specifically, a lot of effort has been devoted to solve the non-connected version of MLP. Thus, previous works provide a good starting point for the development of efficient approaches to solve new versions of MLP.

Recent researches show a growing interest in the use of exact approaches to solve optimization problems in WSN [1,15,25]. Column generation (CG) has been largely used to address different versions of MLP. CG decomposes the problem into a restricted master problem (RMP) and a pricing subproblem (PS). The former maximizes lifetime using an incomplete set of columns, and the latter is used to identify new profitable columns. Gu et al. [16] have studied the coverage and scheduling problem in WSN. As maximum network lifetime problem with coverage constraints inherently involves time issues, the problem is represented by using a time-dependent structure that considers the coverage as a function of time and impose constraints

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on it. Then, they show that this kind of representation for MLP can always be converted into a pattern-based representation that points to maximize lifetime by using subsets of sensors (patterns) that satisfy the coverage requirement. As the number of feasible patterns grows exponentially with the number of sensors, the authors conclude that CG offers a natural way to address coverage and scheduling problems in WSN. Experimental results show that this approach is able to find optimal solutions to medium size instances of MLP. Moreover, recent researches show that this method can be improved by using heuristic approaches embedded in CG to solve the pricing subproblem [25,26].

When the connectivity constraint is also required, the problem is referred to as CMLP. In area coverage, a sufficient condition to guarantee connectivity is that the communication range R_c is at least twice the sensing range R_s ($R_c \geq 2R_s$) [29]. However, Lu et al. [22] have shown that this property does not hold for target coverage, and have proposed a distributed heuristic to solve the problem. Further results presented by Singh et al. [26] show that, for the Q-coverage version of CMLP, in which each target has to be covered by at least Q active sensors at any time, an efficient approach can be generated by relaxing connectivity constraints in the PS. In other words, by solving the problem as in MLP and trying to restore connectivity if necessary.

CMLP has been addressed by Cardei and Cardei [5] who propose three different heuristic approaches. First, an integer programming model of the problem which is solved to create, through a heuristic, an energy-efficient scheme of the problem. Then, the authors propose two greedy heuristics to create iteratively set covers in centralized and distributed manners respectively. Gentili and Raiconi [14] propose a greedy procedure namely CMLP-Greedy and two variants embedded in a greedy randomized adaptive search procedure (GRASP) to find pattern based solutions seeking to maximize network lifetime. Furthermore, the authors compare their results with an exact decomposition based approach using CG and show that their method is computationally efficient and, in addition, is able to find near optimal solutions in most cases.

Zhao and Gurusami [31] propose to solve CMLP by modeling the problem as a maximum cover tree problem (MCTP). In their proposal, the idea is to find a collection of subtrees and timings to maximize network lifetime. The authors show that MCTP is NP-Complete by reduction of 3-SAT problem. The authors propose an upper bound to the network lifetime and propose two heuristics to solve the problem.

Several mixed approaches combining heuristic and exact approaches have been introduced recently to solve optimization problems in various of contexts [4,23]. Heuristic approaches combining CG or Lagrangian relaxation with (meta)heuristic approaches are shown to be successful in a lot of applications. Recently, Rossi et al. [25] presented an efficient implementation of a genetic algorithm based CG to extend lifetime and maximize coverage in wireless sensor networks under bandwidth constraints. The authors show that the use of metaheuristic methods to solve PS in the context of CG allows to obtain optimal solutions quite fast, and to produce high quality solutions when the algorithm is stopped before returning an optimal solution.

In this paper an exact multilevel approach based on CG is proposed to solve the connected maximum network lifetime problem. Our proposal is to speed up the solution process by embedding two heuristic approaches within the CG framework. First, a greedy randomized adaptive search procedure (GRASP) [13] is proposed to solve PS. This approach relaxes connectivity constraints, so a repair procedure is necessary. Then, when the GRASP approach fails to find a profitable solution to PS, a variable neighborhood search (VNS) heuristic [18] is attempted for finding profitable columns. Finally, if both heuristics are unable to find a profitable solution, integer linear programming (ILP) is used to solve PS. It is also used for proving optimality of the current RMP solution at the very end of the search. An extension of the problem, namely α-CMLP, is also considered. It consists in replacing the full coverage requirement by a constraint for enforcing a minimum quality of service. Thus, it is possible to neglect a fraction $1-\alpha$ of the targets, which allows to extend lifetime.

This paper is organized as follows. Section 2 introduces the problem description and the decomposition approach used to solve α-CMLP. A detailed description of the proposed approach is presented in Section 3. The results obtained through the use of the proposed methods and a detailed analysis of the computational experiments are reported in Section 4. Finally, conclusions and future work are presented in Section 5.

2. The maximum network lifetime problem under coverage and connectivity constraints

Consider a set $\mathcal{K} = \{k_1, \dots, k_m\}$ of targets with known locations and a set $\mathcal{S} = \{s_1, \dots, s_n\}$ of sensors deployed to cover the targets. If the distance between a sensor node and a target is less than its sensing range R_s, then this sensor is able to cover the target and an *observation link* exists. The sensor nodes collect and (re)transmit the information to other sensor nodes within their communication range R_c (*communication link*). All the information generated by the targets must be collected by a single sink node r. A sensor is able to send the information to the sink node only if a communication link exists between them, otherwise the information have to be addressed indirectly through a multihop path of sensors.

Let E be the set of all pairs $e(u,v)$ such that a communication link exists between the elements $u, v \in \mathcal{S} \cup \{r\}$ or an observation link exists between the elements $u \in \mathcal{K}$ and $v \in \mathcal{S}$. A feasible cover $C_j \subseteq \mathcal{S}$ is a subset of sensors such that for at least $\lceil \alpha|\mathcal{K}| \rceil$ targets, there exists a communication link $e(u,v)$ between $u \in \mathcal{K},\ v \in C_j$ and there exists a path between the elements of C_j and r. The set of all the feasible covers of $\mathcal{S}$ is denoted by $\Omega = \{C_1, C_2, \dots, C_\ell\}$.

Variable t_j is the time during which cover C_j is used. The α-connected maximum network lifetime problem (α-CMLP) is defined as finding a collection of pairs (C_j, t_j), such that network lifetime, $\sum_{j \in \Omega' \subseteq \Omega} t_j$, is maximized without exceeding the battery capacity b_{s_i} of the sensors s_i.

Let $y_{s_i j}$ be a binary parameter that is set to 1 iff sensor s_i is active in cover C_j. α-CMLP can be formulated as the following linear program:

$$\text{Maximize}: \quad \sum_{C_j \in \Omega} t_j \tag{1}$$

$$\text{Subject to}: \quad \sum_{C_j \in \Omega} y_{s_i j} t_j \leq b_{s_i} \quad \forall s_i \in S \tag{2}$$

$$t_j \geq 0 \quad \forall C_j \in \Omega \tag{3}$$

The objective (1) is to maximize the network lifetime by using a collection of covers C_j that meet the connectivity and coverage constraints. The set of constraints (2) is used to guarantee that battery constraints of the sensors are respected. Constraints (3) are the non-negativity constraints.

2.1. Decomposition approach

The model (1)–(3) is linear and is known to be easy to solve with a linear programming solver [9,14]. By contrast, the enumeration of all the feasible covers C_j is generally impossible, as the number of such covers grows exponentially with the number of sensors $\mathcal{O}(2^{|S|})$ which stresses the need for intelligent strategies to