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The multi-depot vehicle routing problem with heterogeneous vehicle fleet: Formulation and a variable neighborhood search implementation

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ABSTRACT

The multi-depot fleet size and mix vehicle routing problem, also known as the multi-depot routing with heterogeneous vehicles, is investigated. A mathematical formulation is given and lower as well as upper bounds are produced using a three hour execution time of CPLEX. An efficient implementation of variable neighborhood search that incorporates new features in addition to the adaptation of several existing neighborhoods and local search operators is proposed. These features include a preprocessing scheme for identifying borderline customers, a mechanism that aggregates and disaggregates routes between depots, and a neighborhood reduction test that saves nearly 80% of the CPU time, especially on the large instances. The proposed algorithm is highly competitive as it produces 23 new best results when tested on the 26 data instances published in the literature.

1. Introduction

Logistics based companies often use various types of vehicles and operate from more than one distribution center, usually referred to as warehouse or depot. In this situation, the customers are not necessarily assigned to their nearest depots. In this study, we investigate this practical logistical problem where the aim is to minimize the total distribution cost while satisfying the necessary constraints. This problem is known as the multi-depot fleet size and mix vehicle routing problem. This is also referred to as the multi-depot heterogeneous vehicle routing problem (MDHFVRP). This problem seems to suffer from a shortage of published papers when compared to its counterpart namely the standard multi-depot vehicle routing problem (MDVRP) which uses a homogeneous vehicle fleet (i.e., one single type of vehicles only).

Our aim is to revisit and explore further this dormant but practical logistical problem and to provide new best results for future benchmarking purposes. We believe that most companies operate from many depots and use various vehicle types and therefore studying this variant is as useful as other classical variants of the vehicle routing problem (VRP).

In the MDHFVRP, we are given a number of customers, n, a number of depots, m, and a number of vehicle types, K, each of which has a capacity Q_k, a fixed cost F_k and a unit running cost α_k ($k=1,\ldots,K$). In this study, the number of vehicles per type is considered unlimited, each customer must be served by one vehicle only, and each vehicle must start and finish its journey at the same depot. The capacity of any used vehicle and the maximum length of a route must not be exceeded. The objective is to find the least total cost that includes the sum of the vehicle fixed and running costs. As a by-product of solving this problem, the vehicle fleet composition (i.e., the number of vehicles per type) will be found for all depots as well as for each depot.

Though the solution approaches for the single depot and the multi-depot problems have some similarities, it can also be said that it is naive to just decompose the multi-depot problem into several one depot sub-problems by simply assigning the customers to their nearest depots and then apply a suitable approach to each of the single depot problems. Though this simple approach is easy to use, it generally leads to poor suboptimal solutions. Obviously this solution could be considered as a starting solution in other heuristics or meta-heuristics whose search procedure would consider the entire multi-depot problem as one large problem. A brief summary of the MDVRP and its variants is provided in Table 1.

Our aim is achieved through an efficient implementation of a powerful a variable neighborhood search (VNS)-based meta-heuristic. This problem is still a niche and we aim to address which will hopefully attract other researchers to consider studying it. In that paper, the authors put forward a multi-level type heuristic which is similar in principle to the variable neighborhood descent as described in Hansen et al. [14] except that instead of using different neighborhoods, a series of local search operators

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Table 1
Summary of the MDVRP and MDHFVRP related papers.

Authors	Journal (year)	MDVRP/MDHFVRP	Method Used
[42]	Transp Sci (1969)	MDVRP	Clarke and Wright saving-based as above
[43]	Man Sci (1971)	MDVRP	
[46]	ORQuart (1972)	MDVRP	Saving based & refinements
[4]	ORQuart (1972)	MDVRP (School meal delivery problem)	Saving based & refinements
[10]	Omega (1976)	MDVRP	Clustering & sweep heuristic
[13]	Networks (1977)	MDVRP	Borderline customer & saving-based
[1]	Dec Sci (1983)	MDVRP (distribution of chemical product in the USA and Canada)	Saving based & route first, cluster second
[23,32]	Cong Num (1984)	MDVRP	Branch and bound
[3]	Transp Res (1984, 1985)	MDVRP	Incorporate (P) in location routing & formulation
[33]	Working Paper (1986)	MDVRP (delivery to retail outlets from a bakery in Indiana)	Saving method & branch and bound
[23]	AJMMS(1987)	MDVRP	(T–C) modified distance formula & a saving variant
[18]	Transp Sci (1988)	MDVRP	Branch and bound
[27]	IBM Report (1992)	MDVRP (distribution problem of dairy products)	LP & heuristics
[5]	J Bus Log (1992)	MDVRP (distribution problem of the hardware products in the USA)	Exact methods & heuristic
[35]	AJMMS (1993)	MDVRP	Record to record
[39]	COR (1996)	MDVRP	Tabu Search
[6]	EJOR (1997)	MDVRP and MDHFVRP	Multi-level heuristic
[22]	Networks (1997)	MDVRP	Tabu Search
[41]	EJOR (2000)	MDHFVRP with pickups and deliveries	Set covering, & analytic results
[40]	Appl Art Int (2001)	MDVRP	Genetic algorithm & clustering
[45]	J Food Eng (2002)	Open MDVRP (the distribution of meat in Greece)	List-based threshold accepting
[12]	EJOR (2002)	MDVRPTW	One & two stage methods
[31]	J of Heur (2004)	MDVRPTW	VNS
[25]	IEEETASE (2005)	MDVRP with fixed vehicle fleet	Several heuristics
[29]	EJOR (2005)	MDVRP with pickups and deliveries	Combination of a number of heuristics
[34]	COR (2007)	MDVRP	Adaptive large neighborhood search
[9]	EJOR (2007)	MDHFVRP with time windows	Exact methods
[15]	EAAI (2008)	MDVRP	Genetic algorithm
[30]	J of Sched (2009)	MDVSP (vehicle scheduling)	Five heuristics & formulations
[49]	APJOR (2011)	MDVRP with weight related cost	Formulation & scatter search
[48]	JORS (2011)	MDVRP	Ant Systems
[19]	Exp Sys &Appl (2012)	MDVRP with loading cost	VNS

and perturbation schemes are used to reduce the risk of local optimality. The only closest study to the current study is the work by Dondo and Cerda [9] who proposed an algorithm for the multi-depot vehicle fleet mix (MDVFM) with time windows. Their approach is based on a clustering mechanism to reduce the problem size and a mixed integer linear program (MILP) branch and bound solver. For the case of one depot vehicle fleet mix, see the recent papers by Tutuncu [44] and the one by Imran et al. [21] and the references therein. For the case of the classical MDVRP with homogeneous fleet, see Cordeau et al. [7] when time windows are imposed and Pisinger and Ropke [34] and Yu et al. [48] when no time windows are present. For vehicle routing problems in general including contemporary topics and recent challenges in the field of vehicle routing, the reader will find the edited book by Golden et al. [13] to be useful and informative.

The paper is organized as follows. In Section 2, the mathematical formulation is provided followed by an overview of the proposed VNS algorithm, including its main steps in Section 3. A brief explanation of the main steps of the proposed algorithm is covered in Section 4 and a neighborhood reduction scheme, which is incorporated into the search, is described in Section 5. The computational results are reported and analyzed in Section 6. A summary of our findings and highlights of some research avenues that we believe to be worth pursuing in the future are given in the last section.

2. A mixed integer linear formulation for the MDHFVRP

A mixed integer linear formulation of the MDHFVRP which is an extension of the one originally presented in Salhi et al. [38] for the single depot problem is given. Recently, Lee et al. [24] also reproduced a similar formulation for the single depot problem. The proposed formulation is a flow-based formulation instead of a node-based as this type is shown to be promising in solving other vehicle routing related problems, see for instance the work of Yaman [47].

A four index binary variable which identifies the type of vehicle chosen to travel along a given arc and the depot it originates from is used. This guarantees a vehicle to return from the depot it originated from. If this restriction is relaxed, a three index variable can be used instead.

2.1. Parameters

Let

n and m be the number of customers and depots respectively. These are also denoted by nodes.
$(1,..,n+m)$ where the m depots are represented by $n+1,\ldots,n+m$.
q_i is the demand of the ith node $(i=1,..,n+m)$ with $q_i=0 \quad (i=n+1,\ldots,n+m)$.
K is the number of vehicle types.
Q_k is the capacity of the vehicle of type k $(k=1,\ldots,K)$.
F_k is the fixed cost of the vehicle of type k $(k=1,\ldots,K)$.
α_k is the unit running cost of the vehicle of type k $(k=1,\ldots,K)$
D_{ij} is the distance between nodes i and j $(i, j=1,\ldots, n+m)$.

2.2. Decision variables

Let
$X_{ijkd}=1$ if a vehicle of type k $(k=1,\ldots,K)$ traveling along arc (i,j) $(i=1,\ldots,n+m;\ j=1,\ldots,n+m)$ and originating from depot d $(d=n+1, \ldots,n+m)$ is selected, and $X_{ijkd}=0$ otherwise.