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An effective heuristic for multistage linear programming with a stochastic right-hand side

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ABSTRACT

The multistage Stochastic Linear Programming (SLP) problem may become numerically intractable for huge instances, in which case one can solve an approximation for example the well known multistage Expected Value (EV) problem. We introduce a new approximation to the SLP problem that we call the multistage Event Linear Programming (ELP) problem. To obtain this approximation the SLP constraints are aggregated by means of the conditional expectation operator. Based on this new problem we derive the ELP heuristic that produces a lower and an upper bound for the SLP problem. We have assessed the validity of the ELP heuristic by solving large scale instances of the network revenue management problem, where the new approach has clearly outperformed the EV approach. One limitation of this paper is that it only considers randomness on the right-hand side, which is assumed to be discrete and stagewise independent.

1. Introduction

The Multistage Continuous Stochastic Linear Programming (MCSLP) problem corresponds to a linear programming problem with uncertainty in some of its parameters and with several decision stages. For notational simplicity, considering that all the problems here are multistage, we will drop the M and C and will use the shorter form SLP instead. The main feature of the SLP problem is that the uncertain parameters are revealed gradually over time and our decisions should be adapted to this process. The relevance, applications, properties, approaches and solution methods of this problem can be found in [4,17,32], among others. To address optimization problems under uncertainty one can use different approaches such as chance constraint optimization [28], robust optimization [24] and scenario based optimization [4], among others. In this paper we will focus on the last approach.

The SLP problem with a continuous stochastic process is, in general, numerically intractable. To overcome this difficulty one can approximate the original stochastic process by discretizing it. Of course, the quality of the approximation will depend on the quality of the discretization [25]. The discretization process consists in approximating the original stochastic process by a stochastic process with a finite support which can be represented by a scenario tree. Thus, the first difficulty in scenario based optimization corresponds to built a representative and tractable scenario tree. See [9,15], among others. Once a representative scenario tree has been built one can write the so-called deterministic equivalent problem which corresponds to a large scale structured linear programming (LP) problem.

The second difficulty corresponds to solve this LP problem. State of the art optimization software for example IBM ILOG CPLEX Optimizer [16], or CPLEX for short, can be used to successfully solving SLP instances of moderate size. However, for many SLP instances one needs to use alternative methods which can be classified into exact and approximate ones. Exact methods can deal with a large number of scenarios and are based on Lagrangian relaxation [13], Benders decomposition [14] and interior point methods [6], among others. The performance of these optimization methods can be enhanced by using large computing systems (parallel computing, grid computing, etc.) for example in [21]. However, if the number of scenarios becomes too large, exact methods are impractical. In this case, either one solves the SLP problem approximately or one solves an approximation to the SLP problem. This is the case of schemes such as scenario aggregation [19], scenario sampling [33], stochastic dynamic programming scenario refinement [5], approximate dynamic programming [27], and multistage stochastic decomposition [29], among others. However, even an approximated solution of the SLP problem by

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the Sample Average Approximation method requires an exponential number of sampled scenarios in order to attain a reasonable accuracy [31].

In [7] it is presented an iterative procedure, based on constraint aggregation, to fully solve stochastic programming problems with a convex cost and linear constraints. The idea is to reduce the number of constraints in the original problem by replacing them by aggregated constraints, which are certain weights combinations of the original ones. The method generates a sequence of problems with aggregated constraints whose iterates converge to the optimal solution set of the original problem. In contrast, the heuristic method we present is based on the so-called multistage Event Linear Programming (ELP) problem which approximates the SLP one. This approximation is also based on constraint aggregation, however its objective is not to fully solve the original SLP problem but to compute a good suboptimal solution and a good cost lower bound. The second difference is that in [7] the main computations are performed in the space of the original SLP problem whose dimension may be huge, whereas in the approach we present, the main computations are performed in the space of the ELP problem whose dimension is drastically smaller than the original one. The third difference is that our approach uses the probability distribution of the stochastic parameters as the aggregate weights in contrast with [7], which uses the level of infeasibility of the current iterate for a given constraint, as the corresponding aggregate weight.

In this context it is useful to have some cost bound in order to assess the quality of the approximated solution. One of the oldest bounds is the so-called wait-and-see cost lower bound which can be obtained by solving the SLP problem without satisfying the nonanticipativity constraints [4]. Bounds based on Jensen's or on Edmundson–Madansky inequalities can be found in [4]. In [34] Jensen's bound is improved by relaxing certain constraints and associating dual multipliers with them. Another type of bound in stochastic programming is obtained by aggregation of constraints, variables or stages [2,35]. Such approximations are shown to provide bounds if the randomness appears exclusively either in the objective or in the right-hand side (rhs). Kuhn [20] constructs two discrete and stage-aggregated stochastic programs which provide upper and lower bounds on the SLP optimal cost and are numerically tractable. In the framework of the so-called sample average approximation of the SLP problem, one can infer statistical bounds for the SLP solution value as in [30].

One of the most popular approximations to the SLP problem is the multistage *Expected Value* (EV) problem, which replaces the stochastic parameters of the SLP problem by their expected value. To derive the EV problem, the scenario tree associated to the SLP problem, is reduced into a degenerate scenario tree with one scenario. That is, the EV problem approximates the SLP problem by ignoring uncertainty. As an alternative to the EV problem, we introduce the multistage Event Linear Programming (ELP) problem, which also approximates the SLP problem but without ignoring uncertainty. The ELP problem takes into account uncertainty in a simplified way: roughly speaking, it approximates the multistage scenario tree by a sequence of connected two-stage scenario trees (see Section 4). As far as we know such approach has never been proposed in the literature and it could be useful in the cases where the SLP problem was numerically intractable.

In this paper we will concentrate on the SLP problem with randomness appearing exclusively in the rhs of the constraints, which is assumed to be discrete and stagewise independent. In this case, the bound given by the EV problem, corresponds to Jensen's bound. We will prove that the ELP problem also gives a cost lower bound and try to answer the following questions: Is the ELP bound tighter than the EV one? Is the ELP problem tractable? Is it possible to derive good SLP solutions by using the ELP solutions? What is the computational performance of the ELP heuristic in the case of large scale instances? To answer these questions we have used a testbed of large scale instances of the network revenue management problem (up to 393 millions of variables and 357 millions of constraints). The average CPLEX time for the EV, ELP and SLP approaches has been 198, 328 and 1995 s, respectively (CPLEX has failed to solve 22% of the instances when using the SLP approach). The average worst case optimality gap for the EV and ELP approaches, has been 3.62% and 0.45%, respectively.

Thus, the objectives of this paper are to introduce the ELP problem, to study some of its theoretical properties, to compare it to the EV problem, to analyze the computational effort to solve large scale ELP instances and to consider a scheme for deriving a (hopefully good) feasible solution for the SLP problem. With these objectives in mind, in Section 2 we describe the well known SLP problem and the scenario tree structure. In Section 3 we state the EV problem. In Section 4 we introduce the ELP problem and the event spike structure. In Section 5 we state and prove the theoretical results concerning the EV and the ELP bounds. In Section 6 we see an algorithm for obtaining feasible SLP solutions after solving the EV and ELP problems. Finally, in Section 7 we present the computational results of comparing the EV, ELP and SLP solution values in a large testbed of instances of the network revenue management problem that has been chosen as the pilot case to study the effectiveness of the ELP approach.

2. The multistage LP problem with a stochastic right-hand side

The following parameters, indexes and index sets, will be used throughout the paper.

t	Index for stages, $t \in \mathcal{T} = \{1, \dots, T\}$
k	Index for groups of nodes of the scenario tree, $k \in \mathcal{K}_t = \{1, \dots, K_t\} \quad \forall\ t \in \mathcal{T}$. Two nodes are in the same group k if they have the same ancestor node (see Section 2.1) $\quad \forall\ t \in \mathcal{T}$
l	Index for nodes within the same group of nodes, $l \in \mathcal{L}_t = \{1, \dots, L_t\}$ (see Section 2.1) $\quad \forall\ t \in \mathcal{T}$
$\mathcal{T}^+$	Stands for $\{2, \dots, T\}$
$\mathcal{T}^-$	Stands for $\{1, \dots, T-1\}$
$\mathcal{T}\mathcal{K}_t\mathcal{L}_t$	Stands for $\mathcal{T} \times \mathcal{K}_t \times \mathcal{L}_t \ \forall\ t \in \mathcal{T}$

Let us consider the following multistage deterministic LP problem that we name P_{DLP}:

$$\begin{aligned} \min_x \quad & \sum_{t \in \mathcal{T}} c_t^\top x_t \\ \text{s.t.} \quad & A_1 x_1 = b_1 \\ & \sum_{\tau=1}^{t-1} B_{t\tau} x_\tau + A_t x_t = b_t \quad \forall\ t \in \mathcal{T}^+ \\ & x \geq 0, \end{aligned}$$

where c_t is the vector of the objective function coefficients, A_1 and b_1 are the constraint matrix and the right-hand side (rhs) related to stage $t=1$. For all $t \in \mathcal{T}^+$, $B_{t\tau}$ is the constraint matrix of the decision vector x_τ related to stage $\tau < t$, A_t is the constraint matrix of the decision vector x_t and b_t is the rhs corresponding to stage t.

In real life instances, any of the parameters of P_{DLP} may be stochastic. In order to introduce our new approach, we consider a simpler stochastic version of P_{DLP} where the rhs b_t is the only random vector. A stochastic rhs typically reflects uncertainty in supply and/or demand. This is very often the case for problems arising in manufacturing, telecommunications, transportation and