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Optimizing the pricing and replenishment policy for non-instantaneous deteriorating items with stochastic demand and promotional efforts

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ABSTRACT

Retailer promotional activity has become prevalent in the business world. Promotional efforts impact the replenishment policy and the sale price of goods. In this paper, the problem of replenishment policy and pricing for non-instantaneous deteriorating items subject to promotional effort is considered. We adopt a price dependent stochastic demand function in which shortages are allowed and partially backlogged. The major objective is to simultaneously determine the optimal selling price, the optimal replenishment schedule, and the optimal order quantity to maximize the total profit. First, we prove that a unique optimal replenishment schedule exists for any given selling price. Second, we prove that the total profit is a concave function of price. Third, we present an algorithm to obtain the optimal solution and solve a numerical example. Last, we extend the numerical example by performing a sensitivity analysis of the model parameters and discuss specific managerial insights.

1. Introduction

The pricing and replenishment policy problem of deteriorating items has been extensively investigated by researchers. Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility, or loss of marginal value of commodity, which decreases usefulness. The majority of goods such as medicine, volatile liquids, and blood banks, undergo decay or deterioration over time [1]. Ghare and Schrader [2] were the first researchers to consider an optimal replenishment policy for deteriorating items. Goyal and Giri [3] provided an excellent and detailed review of the literatures on deteriorating inventory. In some inventory systems, such as the inventory of fashionable items, the length of waiting time for the next replenishment would determine whether the backlogging will be accepted or not. Therefore, the backlogging rate is variable and dependent on the waiting time for the next replenishment [4]. Abad [5,6] considered a pricing and lot-sizing problem for products with a variable rate of deterioration, which allows shortages and partial backlogging. Dye [7] developed a joint pricing and ordering policy for a deteriorating inventory with partial backlogging, in which the demand is known as a linear function of price. Chang et al. [8] established an economic order quantity (EOQ) model of deteriorating items, which enables a retailer to determine its optimal selling price and lot-sizing policy with partial backlogging and log-concave demand. Bakker et al. [18] have reviewed inventory systems with deterioration since 2001.

In this paper, we consider the stochastic demand function $D(p)+\varepsilon$ and assume that the distribution function of random variable ϵ is deterministic and independent of time. Numerous researchers, such as Federgruen and Heching [15], Petruzzi and Dada [17], Chen and Simchi-Levi [16], Zhang et al. [19], Pang [21], Zhu [22], and Chao et al. [20], have developed models for joint pricing and inventory control using a stochastic demand function.

A common characteristic of these studies is that the deterioration of items in the inventory begins at the instant of their arrival. In fact, the majority of goods have a time span for maintaining quality or original condition, during which no deterioration occurs. Wu et al. [9] defined the phenomenon as "non-instantaneous deterioration." They developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand to obtain a minimum value for the total relevant inventory cost per unit time. In the real world, this type of phenomenon commonly exists for goods such as firsthand vegetables and fruits, which have a short time span for maintaining fresh quality, during which there is almost no spoilage and but subsequent decay of some of the items. For these types of items, the assumption that deterioration begins at the instant of arrival of the stock may cause retailers to create inappropriate replenishment policies due to the overvaluation of the total annual relevant inventory cost. Therefore, the inventory problems for non-instantaneous deteriorating items must be considered in the field of inventory management.

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Ouyang et al. [10] presented an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Yang et al. [11] developed an inventory system for non-instantaneous deteriorating items with price-dependent demand. In their model, shortages are allowed and partially backlogged. Geetha and Uthayakumar [4] proposed an EOQ based model for non-instantaneous deteriorating items with permissible delay in payments. Maihami and Nakhai [23] established joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging, time and price dependent demand. In another study, Maihami and Nakhai [24] considered the effect of delay in payments on the model and extended their previous study. Recently, Shah et al. [25] optimized the inventory and marketing policy for non-instantaneous deteriorating items with generalized deterioration and holding cost rates.

Retailer promotional activity has become prevalent in the business world. For example, Wal-Mart and Costco frequently stimulate the demand for specific types of electronic equipment by offering price discounts. McDonalds and Burger King frequently offer coupons to attract consumers. Other promotional strategies include free goods, advertising, and displays. Promotion policies are very important for the retailer. Research studies such as that by Tsao and Sheen [12] have shown that promotional efforts significantly impact profit.

Krishnan et al. [13] analyzed the coordination of contracts for decentralized supply chains with the promotional efforts of retailers. Taylor [14] considered a situation in which the demand is influenced by the sales efforts of retailer. Tsao and Sheen [12] examined the problem of dynamic pricing, promotion and replenishment policy for a deteriorating item with supplier's trade credit and retailer's promotional efforts.

An appropriate pricing and replenishment policy model for a non-instantaneous deteriorating item, which considers the promotional efforts and stochastic demand, is presented in this paper. In this model shortages are allowed and partially backlogged. The backlogging rate is variable and dependent on the waiting time for the next replenishment. The main objective is to simultaneously determine the optimal selling price, the optimal replenishment cycle time and the order quantity. To the best of our knowledge, this study is the first analysis to jointly consider pricing and replenishment policy for non-instantaneous deteriorating items with partial backlogging, price-dependent stochastic demand and promotional efforts.

The remainder of the paper is organized as follows: In Section 2, assumptions and notations that are used throughout this paper are presented. In Section 3, we establish the mathematical model and the required conditions for obtaining an optimal solution. In Section 4, we present a simple algorithm to obtain the optimal selling price and the inventory control variables. In Section 5, we illustrate a numerical example. We extend some managerial implications based on the sensitivity analysis of the parameters in Section 6. We summarize the study's finding and provide some suggestions for future research in Section 7.

2. Notations and assumptions

The following notations and assumptions are employed throughout the paper:

2.1. Notations

c	the constant purchasing cost per unit
h	the holding cost per unit per time unit
s	the backorder cost per unit per time unit
o	the cost of lost sales per unit
p	the selling price per unit, where $p > c$
θ	the parameter for the deterioration rate of the stock
ρ	the promotional effort
$D(p)+\varepsilon$	the basic demand function
ε	the random variable of the demand function ($E(\varepsilon)=\mu$)
t_d	the length of time in which the product exhibits no deterioration
t_1	the length of time in which no inventory shortage occurs
T	the length of the replenishment cycle time
Q	the order quantity
p^*	the optimal selling price per unit
t_1^*	the optimal length of time in which no inventory shortage occurs
T^*	the optimal length of the replenishment cycle time
Q^*	the optimal order quantity
$I_1(t)$	the inventory level at time $t \in [0, t_d]$
$I_2(t)$	the inventory level at time $t \in [t_d, t_1]$
$I_3(t)$	the inventory level at time $t \in [t_1, T]$
I_0	the maximum inventory level
S	the maximum amount of backlogged demand
PE	the cost of promotional effort
$TP(p, t_1, T)$	the total profit per unit time for the inventory system
TP^*	the optimal total profit per unit time for the inventory system, that is, $TP^* = TP(p^*, t_1^*, T^*)$.

2.2. Assumptions

i. A single non-instantaneous deteriorating item is assumed.
ii. The replenishment rate is infinite, i.e., the retailer can order any quantity of the product.
iii. The lead time is zero.
iv. Shortages are allowed. We adopt the notation used by Abad [5], in which the unsatisfied demand is backlogged and the fraction of shortage backordered is $\beta(x) = (1/(1+\delta x))$, $(\delta > 0)$, where x is the waiting time for the next replenishment and δ is a positive constant. Note that if $\beta(x) = 1$ (or 0) for all x, then the shortage is completely backlogged (or lost).
v. The basic demand function is $D(p)+\varepsilon$, where $D(p)$ is a decreasing and deterministic function of the selling price p and ε is a non-negative and continuous random variable with $E(\varepsilon) = \mu$.
vi. The distribution function of the random variable ε is determined and independent of time. The parameters of the demand function are stationary over the time horizon.
vii. The promotional effort cost $\text{PE}(\rho, D)$ is an increasing function of the promotional effort and the basic demand $\text{PE} = K(\rho-1)^2[\int_0^1 (D(p)+\varepsilon)dt]^\alpha$, where $K > 0$ and α is a constant. We use the form of the cost of the promotional effort PE from Tsao and Sheen [12], which applies to the situation in which both the market demand and the cost of the promotional effort will increase as the promotional effort increases. This form describes the common real relationship among effort, market demand, and promotional cost. $\rho = 1$ if the retailer does not adopt a promotional policy [12].
viii. The promotional effort ρ affects the demand function $\rho(D(p)+\varepsilon)$, that is, when the retailer engages in promotional effort, its demand function is modified by the coefficient ρ. Because ρ is always greater or equal to 1, the demand always increases or remains constant. Thus, if $\rho > 1$ the demand increases and increased revenue from the increased demand exceeds the cost of the promotional effort PE.

3. Model formulation

In this section, we formulate our model. First, we compute inventory levels. Second, the total profit function is formed. Finally, we obtain the optimal solution.