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Arc routing in a node routing environment

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Abstract

We describe a special variant of the vehicle routing problem (VRP), where there are many customers per road segment. This class of VRPs arises in, e.g. mail delivery, and is a borderline case where both arc routing and node routing techniques may be applied for modeling and solving. In a real-world setting, the problem should be modeled so as to incorporate all important constraining factors. We use a simplified node routing model and aggregate customers into supernodes to reduce problem size. A tabu search metaheuristic for the standard node routing-based VRP is then applied to the aggregated version of the problem. Our method is tested both on test instances from the literature as well as on a portfolio of new test instances especially made to fit the problem at hand. Experimental results are reported, showing that aggregation of customers can lead to substantial improvements both in solution time and solution quality in this setting, especially for larger instances.

Keywords: Vehicle routing; VRP; Tabu search; Aggregation

1. Introduction

In the vehicle routing literature there is a distinction between *node routing*, where the transportation tasks are focused on nodes in a road network, and *arc routing*, where the transportation tasks are defined on road segments. Planning of pickup and delivery for a transportation company represent a typical node routing problem, while gritting and snow removal are prime examples of arc routing. Vehicle routing resolution methods differ substantially between node and arc routing. For a more thorough description of arc routing problems, see Dror [1]. Chapter 1 in Toth and Vigo [2] provides an overview of node routing problems.

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We consider vehicle routing with many stops per road segment. This kind of routing problem may combine aspects of node and arc routing, and is known from, e.g. waste collection and newspaper delivery. In problem instances of this kind, all customers are situated along roads in a network. In practice, large instances with many customers arise, and these are often hard to solve to optimality. Aggregation of customers may then be used to reduce problem size, which allows for a larger part of the solution space to be searched.

There is a desire from the industry for better solutions to this kind of problems, in terms of both the time taken to find a solution and the solution quality. In an operational setting, time is often limited, and a decision maker needs a solution to a problem instance within a minute or two, perhaps as a customer is waiting on the telephone. In long-term planning, when more time to solve the problem is available, one may be more interested in solution quality than how quickly a solution can be found. For large problems, with hundreds or thousands of customers, it is often impossible for exact methods to find the solution that provably gives the shortest total length of all vehicle routes or in other ways is regarded optimal, even if days or weeks of computation time is available. This means that in many situations a proven optimal solution is not reachable within the available time, and thus a tradeoff between solution time and solution quality has to be made.

Rich models for the vehicle routing problem (VRP), where constraints to reflect the real-world situation are added to the basic formulations, should normally be used. In this paper we choose a simplified view to focus on how aggregation may be used to reduce problem size in order to get better solutions, while our methods are intended to be used in a rich model setting.

The rest of this paper is divided as follows. The VRP, together with our extensions, is described in Section 2. We describe the use of aggregation to reduce problem size in Section 3 and mathematical models for the VRP in Section 4. A tabu search metaheuristic to solve the problem is presented in Section 5, followed by a description of test instances and test results in Section 6. Conclusions and future work are presented in Section 7.

2. The vehicle routing problem

The VRP deals with the allocation of transportation tasks to a fleet of vehicles, and the simultaneous routing for each vehicle. The problem was first described by Dantzig and Ramser [3]. The VRP is a computationally hard optimization problem with high industrial relevance.

2.1. Classical formulation

The classical VRP is defined on a graph $G=(N, A)$ where $N=\{0, \dots, n\}$ is a vertex set and $A=\{(i, j) : i, j \in N\}$ is an arc set. Vertex 0 is the depot, the other vertices are the customers. The travel cost between customer i and j is defined by $c_{ij} > 0$and d_i is the demand for customer i. The vehicles are usually identical, each with a capacity q. The goal is then to design a least cost set of routes, all starting and ending at the depot, where each customer is visited exactly once. The total demand of all customers on a route must be within the capacity q. This classical formulation is often referred to as the capacitated VRP or CVRP. If $c_{ij} \neq c_{ji}$ for at least one pair of customers, e.g. due to a one-way road, we have an *asymmetric* VRP.