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A Stackelberg hub arc location model for a competitive environment

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ABSTRACT

In this paper, we consider the design of large-scale multiple allocation hub-and-spoke transportation networks in a competitive environment. We adopt a generic hub arc location model that locates arcs with discounted transport costs connecting pairs of hub facilities. Two firms compete for customers in a Stackelberg framework where the leader firm locates hub arcs to maximize its revenue, given that the follower firm will subsequently locate its own hub arcs to maximize its own revenue. We present an optimal solution algorithm that allocates traffic between the two firms based on the relative utility of travel via the competing hub networks. Results for each competing firm with up to three hub arcs show the important role of competition in designing hub-based transportation systems.

1. Introduction

Hub-and-spoke networks play an important role in many transportation systems. These networks provide efficient transportation between many origins and destinations (e.g., cities) via a set of hubs that serve as switching and flow consolidation points, hub arcs that connect two hubs with a discounted travel cost, and access arcs that connect the non-hub nodes and hubs. Hub networks use fewer arcs than in a point-to-point network and thus can reduce transportation costs by exploiting the economies of scale from consolidated flows. Studies on various hub location models have attracted much attention since O'Kelly [45]. Reviews of hub location research include [6,13,16]. Recent research has extended solutions to larger problems [22,23,53] and addressed multiple capacity levels [24], service considerations [11,17,54], dynamic location [20] and stochasticity [21,54]. Some illustrative applications of the wide range of settings for hub location models are LTL (less-than-truckload) trucking [25], truckload trucking [56], high speed rail [7], postal operations [29], liner shipping [31,32], and airlines ([45] is one of the earliest examples).

The vast majority of hub location research has been directed at finding an optimal (or near-optimal) hub network for a single firm to serve a given set of demand specified as flows between many origins and destinations. However, real-world hub-based transportation systems typically operate in a competitive environment where several carriers compete throughout a geographic region. This competition is likely to influence the optimal hub locations and hub network design. With competition, the customers (e.g., freight shippers or individual travelers) must decide which competing carrier(s) to use, and this is typically based on the relative level of service provided and the cost (or fare) charged. Thus, competitive hub models require designing hub networks for each competitor and allocating the demand among the competitors. The objective is usually to maximize the market share captured, where market share may be measured in terms of the percentage of the passengers, freight, revenue, or profit captured. Reviews of competitive location research for general (non-hub) networks include [26,27,52].

Although a variety of hub location models have been studied in the last two decades, studies on competitive hub location problems are not very common. The earliest work is Marianov et al. [43], which formulated a sequential competitive hub median problem on a network. This model assumes that one firm locates p hubs optimally (as in a multiple allocation p-hub median problem [13]) and then the second firm locates p hubs, given the locations of the first firm's hubs, to maximize the flow captured. Wagner [57] provides improved formulations and results for the problems presented in [43] with optimal solutions for up to 50 nodes and 5 hubs. These works allocate customers between the two firms based on the relative *costs* of the OD paths and include both a binary "all-or-nothing" allocation, where all passengers for each OD pair are allocated to whichever firm provides the lowest cost OD path (with ties being allocated to the first firm), and a five-level fractional allocation, where each firm captures 0%, 25%, 50%,

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75% or 100% of the passengers, depending on the relative OD path costs for the two competitors. Eiselt and Marianov [28] extend this line of research by replacing the discrete passenger allocation mechanism with a continuous proportional allocation based on the relative travel time and travel cost of the OD paths of the competitors. They provide solutions with up to five hubs for each competitor using heuristic procedures where the first of the two competing firms locates its hubs either at random or to provide the optimal p-hub median network. More recently, Luer-Villagra and Marianov [39] use a logit function to model competition in hub networks, where the incumbent firm has an optimal network to serve all demand. The new entrant designs a network to maximize its profit, but the incumbent does not act preemptively or react. Allocation of customers is based on price alone using a logit model and all demand is assumed to be served. Solutions are found using an "ad-hoc metaheuristic" based on a genetic algorithm. This work incorporates both price determination and network design; however it uses a heuristic solution procedure and is a sequential location model (as are [28,43,57]) in considering hub location and network design for the new entrant alone.

In contrast to the research mentioned above, we consider the Stackelberg problem and employ an optimal algorithm. Further, we employ the hub arc model rather than assuming the hub-level network is fully connected. The Stackelberg problem is relevant when competitors are aware of each other and one firm (the leader) locates its hubs in anticipation of another firm (the follower) optimally locating its hubs, based on the known locations of the leader's hubs. Thus, the leader seeks to locate its hubs so that its objective is optimized *after* the follower best locates its hubs. The Stackelberg hub location problem is analogous to the Stackelberg location problem on a network introduced by Hakimi [36] and used in several other studies of non-hub facility location (e.g., [47,51]). Sasaki and Fukushima [50] present a continuous Stackelberg hub location model where passenger allocations are determined by a logit function. The results showed that the leader firm may suffer heavy losses if it neglects to consider the competitor's strategies. Sasaki [46] considers a discrete Stackelberg hub location model with flow threshold constraints to ensure that a firm does not carry an unrealistically low level of flow for any OD pair. The model is formulated as a bilevel programming problem where the upper (leader) and lower (follower) problems are binary integer programs. Both [48] and [50] limit OD paths to a single hub stop, so there is no discounted inter-hub travel (i.e., no hub arcs).

Another relevant stream of research is on competitive airline network design models that also consider hub location. The airline research generally employs logit functions to determine market shares for the competitors in idealized hub networks. Adler [1,2] uses a common market share model based on a utility function that incorporates flight frequencies on the least frequent leg of a trip, airfares, whether the trip is direct or not, passenger type (business or non-business), and the frequency elasticity of demand for each type of traveler. Adler and Smilowitz [3] use a logit function with a slightly different form of utility that includes the number of flight legs in a trip. These models incorporate detailed cost functions (see [55]), and sometimes consider profit maximization, so that not all demand is necessarily served. These works differ from the hub location research that focus on locating hubs and designing networks for more generic transportation systems with a simple distance based cost function. In contrast, the airline specific research may address locating a single hub (e.g., an intercontinental gateway airport), with a focus on issues such as airline alliances and mergers [3] and intercontinental service frequencies [41,42], more than hub network design. An alternate game theoretic approach is used by Lin and Lee [38] to solve small competitive problems with up to three candidate hubs by enumerating all hub combinations.

In this paper, we present a general discrete Stackelberg hub location problem using the multiple allocation hub arc location model [14,15] that locates hub arcs whose endpoints are hub nodes rather than the hub median model that locates fully connected hub nodes as in [43,57]. The hub arc model allows OD paths with one or two stops at hubs and helps concentrate flows on the discounted hub arcs, by relaxing the restriction in hub median models that every flow between two hubs is discounted, whether or not it is warranted by the level of flow. This is important to better match the model for economies of scale with actual flows in the solutions [12]. Furthermore, in our model each firm seeks to maximize the revenue (not traffic) captured and we employ a customer allocation mechanism based on the relative utility of the OD paths for each competitor, introduced in [43], to model the different customer behaviors in selecting between the competing carriers. Unlike other models that implicitly treat all customers (e.g., shippers) equivalently by maximizing the traffic captured, in our model customers who generate higher revenues are more valuable. We assume the same revenue applies to each competitor for each OD pair, which may be realistic in the long run due to competitive pressures. This allows us to focus on competition from the different levels of service offered by the OD paths through the hub networks rather than having the conflicting interactions from price competition. This can be a strong assumption in some contexts, such as for passenger airlines which use extremely complex and dynamic pricing policies [18]. However, our goal is not to model a particular network or particular pricing strategies in great detail, but rather to analyze a more general model for hub-based transportation, so as to develop general insights regarding how competition affects the optimal hub locations and network design.

The remainder of this paper is organized as follows. In Section 2, we provide some background and a formulation of the model. Section 3 describes the solution algorithm and Section 4 includes computational results. In Section 5, we give concluding remarks and mention some areas for future work.

2. Model description

In this paper we consider the case when the firms' hub sets are disjoint, so they do not share any hubs or hub arcs (as in [41,42]). This seems most realistic for situations where capacity at a hub (e.g., airport) limits operations to a single firm, as for air express (e.g., FedEx and UPS). It can also be a factor for air passenger transportation as the scale of operations at a large airline's hub (e.g., Atlanta for Delta) tends to limit opportunities for other large airlines to have the high number of operations needed at a hub. However, for trucking it is entirely reasonable to have truck terminals for competitors located in the same city, as the public infrastructure (roads) is usually not a constraint. Note that when competitors are allowed to share hubs, then the follower can capture at least 50% of the market by using the same hub arcs as the leader. Therefore, the more interesting cases are likely to be when the number of hub arcs and/or hubs differ between the leader and follower. Our model and solution algorithm could easily by modified to handle co-located hubs and hub arcs, but we leave these situations for future research. OD paths for our hub arc location model are limited to three arcs, where the first arc is for collection from the origin to a hub, the second one is a central hub arc for transfer between two hubs, and the last arc is for distribution from a hub to the destination. Each of these may be a degenerate arc (from a node to itself) if the node is a hub.

Fig. 1 shows three possible multiple allocation hub networks that serve flows among seven origin/destination nodes. Fig. 1 (a) shows a 3-hub median solution with hubs at nodes 1, 2 and