Computers & Operations Research 47 (2014) 42–52

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Optimal joint replenishment, delivery and inventory management policies for perishable products

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ARTICLE INFO

Available online 11 February 2014

Keywords:
Perishable products
Inventory control
Replenishment
Inventory-routing
Vendor-managed inventory
Branch-and-cut algorithm

ABSTRACT

In this paper we analyze the optimal joint decisions of when, how and how much to replenish customers with products of varying ages. We discuss the main features of the problem arising in the joint replenishment and delivery of perishable products, and we model them under general assumptions. We then solve the problem by means of an exact branch-and-cut algorithm, and we test its performance on a set of randomly generated instances. Our algorithm is capable of computing optimal solutions for instances with up to 30 customers, three periods, and a maximum age of two periods for the perishable product. For the unsolved instances the optimality gap is always small, less than 1.5% on average for instances with up to 50 customers. We also implement and compare two suboptimal selling priority policies with an optimized policy: always sell the oldest available items first to avoid spoilage, and always sell the fresher items first to increase revenue.

1. Introduction

Inventory control constitutes an important logistics operation, especially when products have a limited shelf life. Keeping the right inventory levels guarantees that the demand is satisfied without incurring unnecessary holding or spoilage costs. Several inventory control models are available [3], many of which include a specific treatment of perishable products [30].

Problems related to the management of perishable products' inventories arise in several areas. Applications of inventory control of perishable products include blood management and distribution [5,9,17,18,20,25,26,33], as well as the handling of radioactive and chemical materials [1,11,37], of food such as dairy products, fruits and vegetables [4,12,29,31,35,36], and of fashion apparel [28]. Several inventory management models have been specifically derived for perishable items, such as the periodic review with minimum and maximum order quantity of Haijema [15], and the periodic review with service level considerations of Minner and Transchel [24]. Reviews of the main models and algorithms in this area can be found in Nahmias [30] and in Karaesmen et al. [19]. A unified analytical approach to the management of supply chain networks for time-sensitive products is provided in Nagurney et al. [27].

Efficient delivery planning can provide further savings in logistics operations. The optimization of vehicle routes is one of the most developed fields in operations research [21]. The integration of inventory control and vehicle routing yields a complex optimization problem called inventory-routing whose aim is to minimize the overall costs related to vehicle routes and inventory control. Recent overviews of the inventory-routing problem (IRP) are those of Andersson et al. [2] and of Coelho et al. [8].

The joint inventory management and distribution of perishable products, which is the topic of this paper, gives rise to the perishable inventory-routing problem (PIRP). Nagurney and Masoumi [25] and Nagurney et al. [26] studied the distribution and relocation of human blood in a stochastic demand context, considering the perishability and waste of blood related to age and to the limited capacity of blood banks. Hemmelmayr et al. [16] studied the case of blood inventory control with predetermined fixed routes and stochastic demand. The problem was solved heuristically by integer programming and variable neighborhood search. Gumasta et al. [14] incorporated transportation issues in an inventory control model restricted to two customers only. Custódio and Oliveira [10] proposed a strategical heuristic analysis of the distribution and inventory control of several frozen groceries with stochastic demand. Mercer and Tao [23] studied the weekly food distribution problem of a supermarket chain, without

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http://dx.doi.org/10.1016/j.cor.2014.01.013
0305-0548

considering product age. A theoretical paper developing a column generation approach was presented by Le et al. [22] to provide solutions to a PIRP. The optimality gap was typically below 10% for instances with eight customers and five periods under the assumptions of fixed shelf life and flat value throughout the life of the product.

This paper makes several scientific contributions. We first classify and discuss the main assumptions underlying the management of perishable products. We then formulate the PIRP as a mixed integer linear program (MILP) for the most general case, and we also model it to handle the cases where retailers always sell older items first, and where they sell fresher items first. We devise an exact branch-and-cut algorithm for the solution of the various models. To the best of our knowledge, this is the first time an IRP is modeled and solved exactly under general assumptions in the context of perishable products management. Our models do not require any assumption on the shape of the product revenue and inventory cost functions. We also establish some relationships between the PIRP and the multi-product IRP recently studied by the authors [7].

The remainder of the paper is organized as follows. In Section 2 we provide a formal description of the PIRP. In Section 3 we present our MILP model and its two variants just described, including new valid inequalities. This is followed by a description of the branch-and-cut algorithm in Section 4. Computational experiments are presented in Section 5. Section 6 concludes the paper.

2. Problem description

The joint replenishment and inventory problem for perishable products is concerned with the combined optimization of delivery routes and inventory control for products having a transient shelf life. Here, we consider a three-echelon supply chain in which suppliers deliver products to retailers who then sell products to the end-customers. These products typically have an expiry date, after which they are no longer fit for consumption. This is the case not only of some law-regulated products such as food and drugs, but also of a wide variety of unregulated products whose quality, appearance or commercial appeal diminishes over time, such as flowers, cosmetics, paint, electronic products or fashion items. In this section we discuss four main assumptions underlying the treatment of these kinds of products, and we explain how we incorporate them in our model. Specifically, we discuss the types of product perishability in Section 2.1, the assumptions governing the inventory holding costs of these products in Section 2.2, their revenue as a function of age in Section 2.3, and the management of items of different ages held in inventory in Section 2.4.

2.1. Types of product perishability

There exist two main types of perishable products according to how they decay [30]. The first type includes products whose value does not change until a certain date, and then goes down to zero almost immediately. This is the case of products whose utility eventually ceases to be valued by the customers, such as calendars, year books, electronics or maps, which quickly become obsolescent after a given date or when a new generation of products enters the market. However, this is more a case of obsolescence than perishability. Even though these items may still be in perfect condition, they are simply no longer useful. Within the same category, we find products with an expiry date, such as drugs, yogurt and bottled milk. These products can be consumed whether they are top fresh or a few days old, but after their expiry date, they are usually deemed unfit for consumption. The second type includes products whose quality or perceived value decays gradually over time. Typical examples are fruits, vegetables and flowers. The models introduced in Section 3 can handle both types of products without any ad hoc modification. Raafat [34] describes a stochastic model in which the deterioration is a function of the on-hand inventory level. Our model does not work under the assumption of a random lifetime.

2.2. The impact of item age on inventory holding costs

As a rule, the unit inventory holding cost changes with respect to the age and value of a product. This general assumption holds, for instance, for insurance costs which are value related. All the variable costs related to the age of the product can be modeled through a single parameter, called the unit inventory holding cost, which depends on the age of the item. In some contexts, all items yield the same holding cost, regardless of their age. Products with a short shelf life usually fit in this category. In this case, the holding cost, which encompasses all other variable costs, can be captured by a unique input parameter independent of the value and age of the product, which is the case in most applications.

2.3. Revenue of the item according to its age

A parameter that greatly affects the profit yielded by products of different ages is their perceived value by consumers. Brand new items usually have a higher selling price, which decreases over time according to some function. In this paper we do not make any specific assumption regarding the shape of this function. Rather, we assume that the selling price is known in advance for each product age. Note that the function describing the relation between price and age can be non-linear, non-continuous or even non-convex, but it can still be accommodated by our model, as will be shown in Section 3.

2.4. Inventory management policies

The final assumption we discuss relates to the management of items of different ages held in inventory. It is up to the retailer to decide which items to offer to customers, which will influence the associated revenue. In such a context, three different selling priority policies can be envisaged. The first one consists of applying a *fresh first* (FF) policy by which the retailer always sells the fresher items first. This policy ensures a longer shelf life and increases utility for the customers but, at the same time, yields a higher spoilage rate. The second policy is the reverse. Under an *old first* (OF) policy, older items are sold first, which generates less spoilage, but also less revenue. The third policy, which we introduce in our model, is more flexible and general, and encompasses these two extremes. The *optimized priority* (OP) policy lets the model determine which items to sell at any given time period in order to maximize profit. This means that depending on the parameter settings, one may prefer to spoil some items and sell fresher ones because they generate higher revenues.

Although they are similar, FF and OF policies are different from the traditional FIFO and LIFO policies common in inventory management. Under a FIFO policy, the first product delivered will be the first to be sold. This coincides with an OF policy only if deliveries from the supplier to the retailer is always of fresh items. However, when the supplier delivers products of different ages in different periods, the sequence of deliveries does not necessarily coincide with the ages of the products in inventory. To illustrate, consider the case where the supplier delivers new items on day one, and three-day old items on day two. Then, on day three, different solutions will be obtained under the OF and the FIFO policies. Indeed, under the FIFO policy, the newer items (delivered on day one) will be sold first, but the older items (delivered on day two) will be selected under an OF policy.