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Minimizing total tardiness in a two-machine re-entrant flowshop with sequence-dependent setup times

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ABSTRACT

We consider a two-machine re-entrant flowshop scheduling problem in which all jobs must be processed twice on each machine and there are sequence-dependent setup times on the second machine. For the problem with the objective of minimizing total tardiness, we develop dominance properties and a lower bound by extending those for a two-machine re-entrant flowshop problem (without sequence-dependent setup times) as well as heuristic algorithms, and present a branch and bound algorithm in which these dominance properties, lower bound, and heuristics are used. For evaluation of the performance of the branch and bound algorithm and heuristics, computational experiments are performed on randomly generated instances, and results are reported.

1. Introduction

We consider a two-machine re-entrant flowshop scheduling problem with the objective of minimizing total tardiness. While each job visits each machine once in a typical flowshop, in a re-entrant flowshop, each job should visit the machines two or more times and hence there are re-entrant flows. In this paper, we focus on a case in which all jobs must be processed twice on each machine and there are sequence-dependent setup times on the second machine. Each job should be processed on machine 1 and machine 2, and then on machine 1 and machine 2 again. This scheduling problem can be denoted by $RF2|sdst|T$ in the three-field notation of Graham et al. [1], where $RF2$, $sdst$, and T represent the re-entrant two-machine flowshop, sequence-dependent setup time, and total tardiness, respectively.

Re-entrant flowshops can be found in many manufacturing plants, such as printed circuit board manufacturing systems, semiconductor manufacturing systems, and mirror manufacturing systems [2]. In some of these systems, processing of a job requires a setup operation and setup time for a job may depend on the sequence of the jobs. Such re-entrant flowshops with sequence-dependent setup times can also be found in practical situations. For example, between the etching and implantation workstations of the semiconductor manufacturing systems, there are re-entrant flows and sequence-dependent setup times.

First, we briefly review literature on flowshop problems with tardiness measures. Although there are a large number of research articles on flowshop scheduling problems, relatively fewer researchers have dealt with flowshop problems with due date-related measures. For the two-machine flowshop scheduling problem, Sen et al. [3], Kim [4], and Schaller [5] propose branch and bound (B&B) algorithms with the objective of minimizing total tardiness, whereas Kim [6] and Chung et al. [7] develop B&B algorithms to minimize total tardiness in m-machine flowshops problems.

On the other hand, Gelders and Sambandam [8] present heuristics that are based on priority rules for minimizing the sum of weighted tardiness, and Ow [9] develops a heuristic for a case in which processing times of the jobs on the machines depend on characteristics of the jobs such as order/processing quantities of the jobs. Also, various meta-heuristics have been devised for flowshop problems with tardiness measures. For example, Parthasarathy and Rajendran [10] propose a simulated annealing algorithm to minimize the weighted sum of tardiness and Armentano and Ronconi [11] present a tabu search method, while Onwubolu and Mutingi [12] and Gen and Lin [13] develop genetic algorithms. A review on various heuristics developed for m-machine flowshop problems is given in Vallada et al. [14]. Note that the flowshop scheduling problem with the objective of minimizing total tardiness is shown to be NP-hard even for the two-machine case [15].

There also have been a number of studies on re-entrant flowshop scheduling problems. Graves et al. [16] view a wafer fab as a re-entrant flowshop and give a simple scheduling algorithm for the objective of minimizing average throughput time while meeting a given production rate. For the objective of minimizing maximum lateness, Demirkol and Uzsoy [17] suggest decomposition methods in a re-entrant flowshop. Meanwhile, Choi and Kim

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[2] present a B&B algorithm for minimizing makespan on a two-machine re-entrant flowshop, and Choi and Kim [18] develop heuristic algorithms for an m-machine re-entrant flowshop with the objective of minimizing makespan. In addition, Yang et al. [19] propose a B&B algorithm for two-machine re-entrant flowshops with multi-family jobs and setup times between different families for the objective of minimizing makespan. On the other hand, Choi and Kim [20] present a B&B algorithm to minimize total tardiness of jobs in a two-machine re-entrant flowshop. Note that the re-entrant permutation flowshop scheduling problem with the objective of minimizing makespan is NP-hard in the strong sense even for the two-machine case [21].

In this paper, we consider a two-machine re-entrant flowshop scheduling problem in which there are sequence-dependent setup times on the second machine with the objective of minimizing total tardiness. In this problem, we only consider permutation schedules, in which processing sequences of jobs on the two machines are the same. Although permutation schedules are not dominant in the re-entrant flowshop problem as shown by Choi and Kim [2], permutation schedules are preferred in many real manufacturing systems because of the ease of operation and control of the systems. Also, in some real systems, only permutation schedules are feasible because of inflexibility of material handling systems or limited buffer space.

In this research, it is assumed that: (1) all jobs are available at the beginning of the scheduling horizon, or at time zero; (2) no job can be preempted; and (3) machines do not fail (there is no breakdown of the machines). Since each job should be processed twice on each machine, we consider each pass (through the two machines) of each job as a *sub-job* as in Choi and Kim [2,18,20]. Then, the problem can be reduced to a flowshop problem with $2n$ sub-jobs. However, in this flowshop problem, we have to consider precedence relationship between the first-pass sub-job and the second-pass sub-job of the same job. In other words, the first operation of the second-pass sub-job cannot be started until the completion of the second operation of the first-pass sub-job.

The problem considered in this study can be easily proven to be NP-hard in the strong sense, since the ordinary two-machine flowshop scheduling problem with the objective of minimizing total tardiness is NP-hard [15]. Note that a special case of our problem, in which there is neither sequence-dependent setup times nor re-entrant flows, is the ordinary two-machine flowshop problem. To solve the problem in this study, we develop dominance properties and a lower bound by extending those for a two-machine re-entrant flowshop problem without sequence-dependent setup times. Also, we develop heuristics to find upper bounds, which are list scheduling algorithms and constructive heuristic algorithms by modifying the algorithm developed by Nawaz et al. [22] and the algorithm of Framinan and Leisten [23]. Then, we suggest a B&B algorithm using them.

This paper is organized as follows. In Sections 2 and 3, we develop dominance properties and a lower bound that can be used in a B&B algorithm. In Section 4, we develop a B&B algorithm using these dominance properties and the lower bound, while in Section 5, we propose several heuristics for obtaining feasible solutions. These heuristics are used in the B&B algorithm as well. For the evaluation of performance of the B&B and heuristic algorithms, computational experiments are performed and results are reported in Section 6. Finally, Section 7 concludes the paper with a short summary and discussions on further research.

2. Dominance properties

As stated earlier, in this paper, we present a branch and bound (B&B) algorithm for the scheduling problem under consideration. In this section, we develop dominance properties that can be used in the B&B algorithm. We use the following additional notation.

i	index of jobs (i=1, 2,…, n)
m	index of machines (m=1, 2)
i^k	kth-pass sub-job of job i (i=1, 2,…, n and k=1, 2)
d_{i^2}	due date of sub-job i^2, which is equal to the due date of job i
$p_{i^k m}$	processing time of sub-job i^k on machine m
$\boldsymbol{\sigma}$	partial sequence (schedule), which is to be placed at the front of a complete schedule
U	set of unscheduled sub-jobs, which are not included in partial sequence $\boldsymbol{\sigma}$
$s_{i^k j^{k'}}$	(sequence-dependent) setup time between sub-jobs i^k and $j^{k'}$
$s_{iU}^{\min}$	$=\min_{j\in U} s_{ij}$, i.e., the minimum value of the setup times incurred when jobs in U are processed immediately after job i, or sub-job i^k, k=1 or 2
$s_{iU}^{\max}$	$=\max_{j\in U} s_{ij}$, i.e., the maximum value of the setup times incurred when jobs in U are processed immediately after job i
$s_{Ui}^{\min}$	$=\min_{j\in U} s_{ji}$, i.e., the minimum value of the setup times incurred when job i is processed immediately after jobs in U
$s_{Ui}^{\max}$	$=\max_{j\in U} s_{ji}$, i.e., the maximum value of the setup times incurred when job i is processed immediately after jobs in U
$C_{i^k m}(S)$	completion time of sub-job i^k on machine m in schedule S
$C_m{}^{\circ}(S)$	maximum completion time of sub-jobs in schedule S on machine m, i.e., the time when all sub-jobs in S are completed on machine m
$T(S)$	total tardiness of second-pass sub-jobs in schedule S
$\boldsymbol{\pi}$	arbitrary (partial) sequence of sub-jobs that are not included in $\boldsymbol{\sigma}$
$\boldsymbol{\sigma} i^k \ldots j^{k'}$	partial sequence (schedule) obtained with $\boldsymbol{\sigma}$ followed by sub-jobs i^k,…, $j^{k'}$ in this order
$\boldsymbol{\sigma} i^k \ldots j^{k'} \boldsymbol{\pi}$	partial sequence (schedule) with $\boldsymbol{\sigma} i^k \ldots j^{k'}$ followed by $\boldsymbol{\pi}$

The following propositions give dominance properties, which can be used to identify partial schedules that are dominated by others or those that dominate others. A partial schedule, $\boldsymbol{\upsilon}$, is said to be *dominated* if there is another partial schedule that results in a complete schedule better than the best possible complete schedule resulting from $\boldsymbol{\upsilon}$. Thus, when a node (corresponding to a partial schedule) is generated in a B&B algorithm, it can be pruned if its corresponding partial schedule is dominated by another partial schedule. The dominance properties to be presented in this paper may be considered as extended versions of those presented in Schaller [5] and Choi and Kim [20] for two-machine flowshop tardiness problems. In such extended versions, sequence-dependent setup times are to be considered in dominance conditions.

First, in the following four propositions, we develop dominance properties related to two adjacent sub-jobs that are to be placed last in a partial sequence. Proposition 1 gives a dominance condition related to a first-pass sub-job and a second-pass sub-job. Note that this proposition is extended from Proposition 3 of Choi and Kim [20].

Proposition 1. *Given a partial schedule,* $\boldsymbol{\sigma}$, *if sub-jobs* i^1 *and* j^2 *such that* $i^1 \notin \boldsymbol{\sigma}$, $j^2 \notin \boldsymbol{\sigma}$, *and* $j^1 \in \boldsymbol{\sigma}$, *satisfy* (a) $C_2{}^{\circ}(\boldsymbol{\sigma} i^1 j^2) + s_{jU}^{\max} \le C_2{}^{\circ}(\boldsymbol{\sigma} j^2 i^1) + s_{iU}^{\min}$ *and* (b) $C_2{}^{\circ}(\boldsymbol{\sigma} i^1 j^2) \le d_{j^2}$, *then* $\boldsymbol{\sigma} j^2 i^1$ *is dominated.*

Proof. We show that $T(\boldsymbol{\sigma} i^1 j^2 \boldsymbol{\pi}) \le T(\boldsymbol{\sigma} j^2 i^1 \boldsymbol{\pi})$ for any sequence $\boldsymbol{\pi}$ that makes $\boldsymbol{\sigma} i^1 j^2 \boldsymbol{\pi}$ and $\boldsymbol{\sigma} j^2 i^1 \boldsymbol{\pi}$ complete sequences. Since $C_1{}^{\circ}(\boldsymbol{\sigma} i^1 j^2) = \max\{C_1{}^{\circ}(\boldsymbol{\sigma}) + p_{i^1 1}, C_{j^1 2}(\boldsymbol{\sigma})\} + p_{j^2 1}$ and $C_1{}^{\circ}(\boldsymbol{\sigma} j^2 i^1) = \max\{C_1{}^{\circ}(\boldsymbol{\sigma}), C_{j^1 2}(\boldsymbol{\sigma})\} + p_{j^2 1} + p_{i^1 1} = \max\{C_1{}^{\circ}(\boldsymbol{\sigma}) + p_{i^1 1}, C_{j^1 2}(\boldsymbol{\sigma}) + p_{i^1 1}\} + p_{j^2 1}$, we have $C_1{}^{\circ}(\boldsymbol{\sigma} i^1 j^2) \le C_1{}^{\circ}(\boldsymbol{\sigma} j^2 i^1)$. Also, tardiness of job j (or sub-job j^2) in $\boldsymbol{\sigma} i^1 j^2$ is 0 from condition