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Minimizing the weighted sum of maximum earliness and maximum tardiness costs on a single machine with periodic preventive maintenance

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A R T I C L E I N F O

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A B S T R A C T

We consider the problem of scheduling a set of jobs on a single machine against a common and restrictive due date. In particular, we are interested in the problem of minimizing the weighted sum of maximum earliness and maximum tardiness costs. This kind of objective function is related to the just-in-time environment where penalties, such as storage cost and additional charges for late delivery, should be avoided. First we present a mixed integer linear model for the problem without availability constraints and we prove that this model can be reduced to a polynomial-time model. Secondly, we suppose that the machine undergoes a periodic preventive maintenance. We present then a second mixed integer linear model to solve the problem to optimality. Although the latter problem can be solved to optimality for small instances, we show that the problem reduces to the one-dimensional bin packing problem. Computational results show that the proposed algorithm best fit decreasing performs well.

1. Introduction

Sequencing and scheduling decisions are crucial in manufacturing industries especially in a just-in-time environment (JIT). Indeed, companies have to meet delivery dates that have been committed to customers and subsequently they have to mediate between two conflicting objectives, namely early delivery and late delivery. Just-in-time manufacturing is a process that continuously stresses waste reduction by optimizing the processes and procedures necessary to maintain a manufacturing operation. The importance of the JIT approach has led to a wide range investigation of scheduling problems that include both earliness and tardiness penalties [1]. In contrast to earlier studies, most recent studies integrate tardiness penalties as well as earliness penalties. Thus, they take into consideration penalties due to delivery after a contractually arranged due date and those due to storage costs, insurance, theft, perishing, and bounded capital. The simplest earliness-tardiness problem involves minimizing the absolute or squared deviations of job completion times from a common due date [2–4]. Given that manufacturing systems constitute the vast majority of company's investment and constitute their production tools, they must be in perfect conditions whenever needed. Especially in a just-in-time environment where customer requirements in terms of quality, quantity, and delay are crucial for competitiveness. Unfortunately, these systems are subject to random failures and to deterioration and therefore have to undergo corrective maintenance. Systems can also be stopped for preventive maintenance reasons. However taking into account maintenance constraints in scheduling problems is not systematic. Many authors assume that the production system is available whenever needed. Obviously this is not the case all the time as there are many reasons why machines may not be in operation. Some of these reasons are based on a deterministic process, others on a random process [1]. When unavailability periods are considered, there are few researchers that explicitly try to integrate preventive maintenance and scheduling decisions on a single machine. For instance the authors of [5] consider the problem of minimizing weighted completion time and they take into consideration only one preventive maintenance period. Ji et al. [6] consider the same problem with the objective of minimizing the makespan. Wang et al. [7] consider the problem of minimizing the total weighted job completion times on a single machine with availability constraints. They show that the problem is NP-hard in the strong sense. However, they propose heuristics for the special case when the weights are proportional and when there is only a single availability constraint. Kacem et al. [8] consider the same objective with one unavailability period. They propose branch and bound algorithm and a dynamic programming to solve exactly such

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a problem. They carried out extensive computational experiments using these approaches, and showed that problems with up to 3000 jobs, can be solved within a reasonable computation time. Later, Kacem and Chu [9] have improved these results by proposing a branch-and-bound algorithm based on a set of improved lower bounds and three heuristics. Recently, Low et al. [10] have addressed the same problem to minimize the makespan where the unavailability of machine results from periodic maintenance activities. Each maintenance period is scheduled after a periodic time interval and the machine should stop to maintain after a periodic time interval or to change tools after a fixed amount of jobs processed simultaneously. They show that this problem is NP-hard in the strong sense and give some heuristic algorithms to solve it. Computational results provided by the authors show that the algorithm first fit decreasing performs well. An excellent survey on scheduling with deterministic machine availability constraints can be found in the paper of Ma et al. [11]. In this survey, authors present recent main complexity results concerning the joint scheduling of production with unavailability periods in single machine, parallel machine, flow shop, open shop and job shop environment.

In this paper we consider the problem of minimizing the weighted sum of maximum earliness and maximum tardiness costs on a single machine under maintenance constraints. As far as we know, this kind of objective function was only studied under the condition of equal unitary weights and without any consideration of maintenance constraints. Amin-Nayeri and Moslehi, as cited in [12], propose a branch-and-bound method to solve a single machine sequencing problem, in which the objective function is to minimize the maximum earliness and tardiness. Tavakkoli-Moghaddam et al. [13] consider the same objective function with general due dates. They use an idle insert algorithm and illustrate its efficiency by solving problems with different job sizes. They report also the optimal value associated with the special case of a common due date. In a subsequent paper Tavakkoli-Moghaddam et al. [14] present two methods to solve the above problem. Based on several experimental results, they show that the proposed simulated annealing metaheuristic has a small error and a lower computational time than the branch-and-bound method. Recently, Moslehi et al. [15] consider the minimization of the sum of maximum earliness and maximum tardiness in a flow shop configuration. They present optimal scheduling in a two-machine flow shop ($n/2/P/ET_{max}$) and determine dominant set for any optimal solution. They use a branch-and-bound method to solve the problem and introduce a number of lemmas to develop an effective algorithm which solve to optimality more than 82% of a selection of problems.

The remaining of the paper is organized as follows: notations and problem description are given in Section 2. In Sections 3 and 4 we will present respectively the problem without availability constraints and the problem with availability constraints. It is in this case the maintenance constraints. The proposed mathematical models in these sections and the properties of the two problems will be discussed extensively. Numerical results are discussed in Section 5. In the final section, conclusions and perspectives are presented.

2. Problem description

We consider the following single machine scheduling problem: n independent jobs are simultaneously available for processing at time zero, and their processing times p_i for $i \in N = \{1, 2, \ldots, n\}$, are known and fixed in advance. All jobs have the same restrictive due date d, with $d \le \sum_{i \in N} p_i$, and are to be processed non-preemptively. Furthermore the machine can handle only one job at any time. A good introduction to common due-date problems can be found in the paper of Baker and Scudder [16]. We define furthermore C_i as the completion time of job $i \in N$ and $P = \sum_{i=1}^{n} p_i$. If C_i is smaller than or equal to the common due date d, the job earliness is $E_i = d - C_i$. Accordingly, job i is tardy with the tardiness $T_i = C_i - d$, if its completion time is greater than the common due date d. As it is not known in advance whether a job $i \in N$ will be completed before or after the due date, earliness and tardiness are calculated as $E_i = \max\{d - C_i, 0\}$ and $T_i = \max\{C_i - d, 0\}$. The maximum earliness E_{max} and the maximum tardiness T_{max} are defined as follows: $E_{max} = \max_{i \in N}\{E_i\}$ and $T_{max} = \max_{i \in N}\{T_i\}$. The objective is to jointly minimize the weighted sum of maximum earliness and maximum tardiness penalties

$$f = \alpha E_{max} + \beta T_{max} \tag{1}$$

where α, β, E_{max} and T_{max} are respectively the per unit time earliness penalty, the per unit time tardiness penalty, the maximum earliness and the maximum tardiness. This problem will be denoted as $1|d_i = d^{res}|ET_{max}$. Practical interest of the problem was given in detail by Tavakkoli-Moghaddam et al. [13]. For instance, the authors cite the case when all jobs done on machines exit from a firm as batches built-up many parts, and the case of an assembly line.

First we study the problem without any availability consideration; this problem will be denoted as P1. This problem can be modeled easily using one of the techniques reported in [17]. Namely models can be based on different decision variables: (i) completion time variables (ii) time index variables (iii) linear ordering variables and (iv) assignment and positional date variables. In the second model, we suppose that the machine availability is affected by the periodic maintenance activities. In other words, the machine must stop product processing from time to time to conduct a preventive maintenance action. This problem will be denoted by P2.

3. Scheduling without availability constraints

3.1. Mixed integer problem formulation

In this section we propose a mixed integer linear problem formulation (MIP1) for the Problem P1 based on the completion time variables. It can be used to obtain optimal solution of the Problem P1:

$$\min \quad f = \alpha E_{max} + \beta T_{max}$$

$$\text{s.t.} \quad C_i \le C_j - p_j + M(1 - x_{ij}) \quad \forall i, j \in N, \; j > i \tag{2}$$

$$C_j \le C_i - p_i + M x_{ij} \quad \forall i, j \in N, \; j > i \tag{3}$$

$$T_i - E_i = C_i - d \quad \forall i \in N \tag{4}$$

$$C_i \ge p_i \qquad \forall i \in N \tag{5}$$

$$E_{max} \ge E_i \quad \forall i \in N \tag{6}$$

$$T_{max} \ge T_i \quad \forall i \in N \tag{7}$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in N, \; j > i \tag{8}$$

$$T_i, E_i \ge 0 \quad \forall i \in N. \tag{9}$$

In this model, the binary variable $x_{ij} = 1$ if the job i is sequenced before the job j, and *0* otherwise, the parameter M is a sufficiently large scalar (e.g. an upper bound on the optimal makespan, the value $\sum_{i \in N} p_i$ is a valid upper bound). Eq. (1) represents the objective function to be minimized. Constraints of form (2) and (3) indicate that no two tasks i and j scheduled on the same machine cannot overlap in time: if the variable $x_{ij} = 1$, the