Computers & Operations Research 44 (2014) 66–74

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Exact formulations and algorithm for the train timetabling problem with dynamic demand

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ARTICLE INFO

Available online 13 November 2013

Keywords:
Train timetabling
Dynamic demand
Regular timetable
Exact algorithm
Branch-and-cut

ABSTRACT

In this paper we study the design and optimization of train timetabling adapted to a dynamic demand environment. This problem arises in rapid train services which are common in most important cities. We present three formulations for the problem, with the aim of minimizing passenger average waiting time. The most intuitive model would consider binary variables representing train departure times but it yields to non-linear objective function. Instead, we introduce flow variables, which allow a linear representation of the objective function. We provide incremental improvements on these formulations, which allows us to evaluate and compare the benefits and disadvantages of each modification. We present a branch-and-cut algorithm applicable to all formulations. Through extensive computational experiments on several instances derived from real data provided by the Madrid Metropolitan Railway, we show the advantages of designing a timetable adapted to the demand pattern, as opposed to a regular timetable. We also perform an extensive computational comparison of all linear formulations in terms of size, solution quality and running time.

1. Introduction

The railway planning process is a complex activity which is usually decomposed into a succession of stages, including network design, line design, scheduling, timetabling, rolling stock, and personnel planning [1,15,17,18]. Timetabling design consists of determining departure and arrival times for each train service to and from each station along a railway line. A service is defined as a trip from an origin to a final destination station. In this paper, a train refers to the service it operates. We consider the case of a double direction rapid transit line with two tracks, in which case departure and arrival times can be designed without train conflicts on track segments, i.e., on the line portions between two consecutive stations.

Timetables are often constructed subject to a regularity or periodicity constraint, using a constant origin–destination peak-hour demand matrix [22,23,29]. Regular timetables are mainly used in rapid transit systems, where the frequency of the train services is high and their departures are equally spaced throughout the planning horizon, for example, every seven minutes. A periodic timetable repeats itself at every period of the planning horizon, for example, trains may be scheduled to depart at 3, 21 and 46 min every hour. Periodic timetables have proved their ability to deal with large-scale railway networks [22], they are easily memorized by passengers and, in the case of constant demand, they yield minimum waiting times [23]. Periodic solutions were initially proposed by Voorhoeve [29] who followed the periodic event scheduling problem (PESP) formulation of Serafini and Ukovich [28].

We now describe some of the main scientific contributions available in this area. In Nachtigall and Voget [27], the authors present a genetic algorithm which is combined with a greedy heuristic and a local improvement procedure to obtain timetables while minimizing the average waiting time. Liebchen and Möhring [24] model the periodic event scheduling problem (PESP) as a digraph in which temporal restrictions on the arcs relate periodically recurring events. In Liebchen and Peeters [25], the authors introduce the concept of integral cycle bases for characterizing periodic tensions, following the work of Nachtigall [26]. Chierici et al. [14] study the quality of timetables and the corresponding demand captured by means of a logit model which computes the modal split between railway and an alternative transportation mode. Cordone and Redaelli [16] develop a branch-and-bound algorithm based on a piecewise-linear approximation of a non-

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http://dx.doi.org/10.1016/j.cor.2013.11.003

convex objective function. These authors presented computational results concerning both random instances and a real-world regional network located in Northwestern Italy. Kroon et al. [20,21] study the problem of improving periodic timetables in the Netherlands under a regular demand assumption. These authors developed a stochastic optimization model to allocate buffer times with the objective of minimizing random disturbances. Some authors deal with the problem of relaxing periodicity in high dense networks, defining flexible time slots for the departure and arrival times instead of exact times in order to achieve feasibility [7], or modelling the problem of finding a feasible partial periodic timetable from an initial PESP formulation [8].

If demand cannot be assumed to be constant over time, the problem then becomes much more general. A periodic timetable applied to a general demand case leads to low occupancy levels of the trains and high average waiting times [9,23]. Non-periodic timetabling is particularly appropriate in long corridors with high track densities.

Caprara et al. [13] use an integer linear programming (ILP) model to determine trains timetable considering modifications over an ideal timetable provided by the train operator. Departure time of trains at the first station can be modified, trains can be cancelled, and speeds and dwell times can be reduced in order to satisfy track capacity constraints. The model incorporates manual block signalling for managing a train on a track segment and maintenance operations that can block a track segment for a given period. Cacchiani et al. [5] extend this ILP model and apply a Lagrangian heuristic algorithm to deal with additional real-world constraints. Cacchiani et al. [4] consider a similar problem in which solutions are forced to satisfy the track capacity constraints while minimizing deviations of departure and arrival train times with respect to an ideal known timetable. They use an ILP formulation, which is obtained from a so-called compatibility graph. Cacchiani et al. [3] consider an alternative ILP model in which each variable corresponds to a full train timetable. They propose heuristic and exact algorithms based on the solution of the LP relaxation model. Ingolotti et al. [19] implement a metaheuristic considering a set of realistic safety and operative constraints. The authors do not consider passenger demand. Their objective is to minimize the deviation between the train delays with respect to the minimum total running time. For a more detailed review on railway timetabling see the recent work of Cacchiani and Toth [2].

In this paper, we focus on constructing timetables adapted to a dynamic demand pattern [10]. Our study is motivated by a collaboration with the Madrid Metropolitan Railway, which provided real demand data for their C5 line. We introduce four different formulations to model this problem. One of the main features of these formulations is that they do not assume any shape for the demand function; they can deal with non-monotonic and even nonconvex demand functions. The objective of the problem is to minimize passenger waiting times at stations. We propose exact algorithms to optimize the models. The solutions are train timetables adjusted to a dynamic demand pattern over a finite planning horizon and are not necessarily regular, nor periodic. We believe this is the first exact algorithm ever proposed for this problem. We note that the train timetabling problem is NP-hard [6,11], which justifies looking for tight formulations and efficient algorithms.

The remainder of this paper is organized as follows. In Section 2 we formally describe the problem and introduce some notation common to all models. In Section 3 we propose three linear mathematical formulations for the problem, followed by the description of a branch-and-cut algorithm applicable to all models in Section 4. We present the results of extensive computational experiments in Section 5, followed by conclusions in Section 6.

2. Problem description

Train timetables are normally represented in form of time-space diagrams as shown in Fig. 1. The x-axis represents the planning horizon, and the y-axis the stations of the considered line, more concretely, the distance from each station to the first one. Fig. 1(a) illustrates a regular timetable, i.e., the headway between consecutive trains is constant, and Fig. 1(b) a non-regular timetable, i.e., headways are not necessarily constant and train frequency is normally higher around peak hours.

We now formally describe the train timetabling design problem for a two-track railway line, one in each direction. The determination of the timetable can then be decomposed into two independent problems. Let $\mathcal{S}=\{1,\dots,n\}$ be the ordered set of stations defining a two-track railway line. The planning horizon is discretized into time intervals of length δ. Thus, time instant $t\in\mathcal{T}=\{0,1,\dots,p\}$ corresponds to δt time units elapsed since the beginning of the planning horizon. The discretization constant δ represents the length of the smallest time interval considered in the problem and so, from now on we will consider it as the time unit which can be as small as desired. Let d_{ij}^{t} be the passenger demand between stations $i,j\in\mathcal{S},\ j>i$ during the interval $[t-1,t]$. We assume that passenger arrival data are available for each time interval. This demand description is very common in modern transit systems where data acquisition devices are installed at the entrance of stations and these data are used to compute the origin–destination matrices. Let l_{ij} be the length of the segment between stations i and j, h_{min} be the minimum headway, i.e., the minimum amount of time required between the departure of two consecutive trains at each station, w_{min} and w_{max} be the minimum and maximum allowed dwell time at stations, and s_{min} and s_{max} be the inverse of the minimum and maximum traveling speed of a train. Note that we work with the inverse of the speeds to avoid non-linear terms in the constraints of the problem.

The aim of the problem is to determine train departure times at stations and train speeds on segments such that the average waiting time of passengers on the stations is minimized.

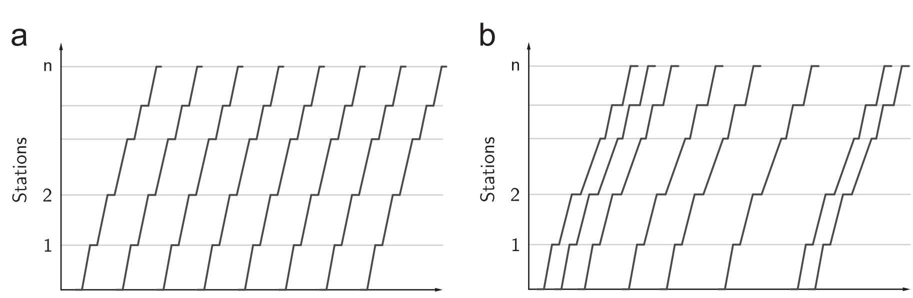

Fig. 1. Time–space diagrams of train timetables for a one line corridor. (a) Regular timetable and (b) non-regular timetable.