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Models and strategies for efficiently determining an optimal vertical alignment of roads

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ABSTRACT

Selecting an optimal vertical alignment while satisfying safety and design constraints is an important task during road construction. The amount of earthwork operations depends on the design of the vertical alignment, so a good vertical alignment can have a profound impact on final construction costs. In this research, we improve the performance of a previous mixed-integer linear programming model, and we propose a new quasi-network flow model. Both models use a piecewise quadratic curve to compute the minimum cost vertical alignment and take earthwork operations into account. The models consider several features such as side-slopes, and physical blocks in the terrain. In addition to improving the precision, we propose several techniques that speed up the search for a solution, so that it is possible to make interactive design tools. We report numerical tests that validate the accuracy of the models, and reduce the calculation time.

1. Introduction

Road design refers to the problem of connecting two given end points by selecting an economical alignment while satisfying various design specifications, safety constraints, and considering environmental and socioeconomic impacts [15,14]. Minimizing the cost of road construction spans a wide range of road types: from highways to mountain roads. It is a subproblem in forest management [4], and in the design of efficient highway networks [3].

The problem of designing roads can be split into three interrelated sub-problems [1]. First, the horizontal alignment provides the road trajectory from a satellite's eye view considering political and social issues [16]. Then, the vertical alignment is a modification of the ground profile that minimizes the cost of construction while satisfying safety and regulation constraints [23]. Finally, a solution to the earthwork problem describes how material is rearranged optimally to build the desired vertical alignment at minimal cost [13]. For computational efficiency, many researchers combine the last two stages together to design optimal vertical alignments while minimizing earthwork costs.

Since two given points can be connected in numerous possible ways, selecting the best alignment quickly from many potential alignments is a difficult task. Manual processes of designing roads take hours of skilled labor and there is no guarantee on the optimality of the final design. In this paper we propose two new models for the vertical alignment problem (incorporating the earthwork problem) that can be solved by deterministic optimization algorithms.

Nondeterministic methods to compute a vertical alignment were considered for example by Lee and Cheng [20] who proposed a three layered heuristic method for vertical alignment optimization, and by Fwa et al. [7] who presented a genetic algorithm formulation of the vertical alignment problem. In [11], Goktepe et al. proposed a vertical alignment model using genetic algorithms. A detailed model solved with a genetic algorithm can be found in [14].

Several deterministic algorithms were proposed that take different constraints into account. Easa [6] proposed a trial and error method by enumerating all possible combinations of feasible grades for vertical alignment considering earthwork cost. Goh et al. [8] used dynamic programming to compute an optimal vertical alignment. Moreb [22] proposed a model combining vertical alignment and earthwork operations in a single linear program that outputs the alignment as a piecewise-linear function. Moreb and Aljohani [24] improved the previous work of Moreb [22] by representing the road profile as a quadratic spline. Goktepe et al. [10] proposed a dynamic programming model to solve the vertical alignment problem, and later in [9] they extended the previous model by combining it with the Weighted Ground Line Method for cut-fill balancing and earthwork minimization. Moreb [23] also proposed an improved linear

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program that solved the problem of sharp connectivity of piecewise linear segments by adding more constraints. Koch and Lucet [19] improved Moreb's model [23] by reducing the error in the slope constraint. Hare et al. [13] were the first to propose a mixed integer linear programming model for earthwork optimization that handles blocks. In [12], Hare et al. proposed a mixed integer linear programming model for vertical alignment considering side slopes and blocks. Ahuja et al. [2, p. 1213] give a network flow model for earthwork optimization without blocks.

In this research, we extend the previous model [12] and propose a new quasi-network flow model based on [2, p. 1213] for the vertical alignment optimization problem considering earthwork costs while satisfying industry standards and rules (supplied by our industry partner Softree Technical Systems Ltd.[1]). This quasi-network flow model incorporates side slopes, physical blocks and other design constraints of the mixed integer linear program proposed by Hare et al. [12]. Using a test set of 60 roads, we numerically examine several techniques to reduce computation time. Our tests clearly demonstrate the speed advantage of the new quasi-network flow model presented in this work. Since the quasi-network flow model puts restrictions on the cost function, we also report several techniques to speed up the search for a solution of the mixed-integer linear programming model.

In Section 2, we define the variables for both models. Then, in Section 2.1, we describe and improve the previously designed mixed integer linear programming model (that we name the *Complete Transportation Graph* (CTG) model) for vertical alignment optimization. In Section 2.2, we introduce the new *Quasi-Network Flow* (QNF) model for vertical alignment optimization. We describe several techniques to improve the solution time in Section 3. In Section 4, we show numerical results and deduce the most efficient techniques. In Section 4.3, we discuss the results and the drawbacks of the models considered. Section 5 concludes the paper with directions for future research.

2. Model description

In this section we describe two models that solve the vertical alignment problem for a known horizontal alignment. The variables, parameters, and sets will be defined as they appear.

In both models, the road profile is designed as a quadratic spline. The idea of approximating the road profile by a spline was described in [24,23]. Later in [19], it was shown that up to a quadratic spline, the linearity of the model can be maintained. In Fig. 1, a ground profile and a vertical alignment of a sample road are shown.

In order to model the vertical alignment, we split the road into several sections. Let $\mathcal{S}=\{1,2,\dots,n\}$ be the index set for the sections. Every section i ($\in \mathcal{S}$) has a station number s_i and an effective length d_i. The difference between a station and a section is that a station is a point, whereas a section is a piece of ground stretched by an associated effective length. At every station, a cross-section area of the ground profile is taken. So we can determine the cross-section area of each material at each station point. The effective length is used to approximate the volume of materials by multiplying with the cross-section area. A section is associated with the volume of materials, whereas a station is associated with the cross-section areas at a particular point.

The height of the ground profile of section i is named h_i. The decision variable u_i is the vertical offset from the ground profile at section i. We assume that throughout the entire length of the section, the road has a fixed height and a fixed cross-section.

[1] http://www.softree.com

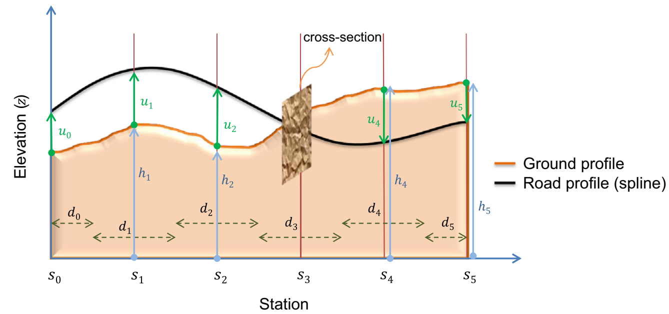

Fig. 1. Side view of ground profile and road profile. Here, d_i is the effective length of a section, h_i is the ground elevation, u_i is the offset from the ground profile. Cross-section areas are given at each station s_i.

In reality the heights and cross-sections vary throughout the section, so this provides only an approximate value. The approximation error can be reduced by decreasing the section length, i.e., increasing the number of sections. However, if section lengths are small, the problem size becomes large.

The entire road profile is represented as a quadratic spline (Fig. 2). The sections are grouped into segments. In every segment there may be several sections, and each segment is associated with a quadratic function. Let $\mathcal{G}=\{1,2,\dots,\overline{g}\}$ be the index set of the segments and $\mathcal{S}_g=\{1,2,\dots,n_g\}$ be the index set of n_g sections in spline segment g. The function $\delta:\mathcal{G}\times\mathcal{S}_g\to\mathcal{S}$ maps the index in the spline segment to the actual section index, i.e., if $\delta(g,j)=i$, then $s_i=s_{\delta(g,j)}$, for all $g\in\mathcal{G},\ j\in\mathcal{S}_g,\ i\in S$.

Every two consecutive segments have one common station between them (Fig. 2). Therefore, the last station of a spline segment is equal to the first station of the next spline segment, i.e., $s_{\delta(g,n_g)}=s_{\delta(g+1,1)},\ \ \forall g\in\{1,\dots,\overline{g}-1\}$. The quadratic function for each segment $g\in\mathcal{G}$ is defined as

$$P_g(s)=a_{g,1}+a_{g,2}s+a_{g,3}s^2,$$

where s is the distance from the current station to the beginning of the road along the alignment. The derivative of $P_g(s)$ is written as

$$P'_g(s)=a_{g,2}+2a_{g,3}s.$$

In general, we can represent the spline function as follows:

$$P(s)=\begin{cases} P_1(s) & \text{if } s_{\delta(1,1)}\le s\le s_{\delta(1,n_1)},\\ P_2(s) & \text{if } s_{\delta(2,1)}\le s\le s_{\delta(2,n_2)},\\ \vdots & \\ P_{\overline{g}}(s) & \text{if } s_{\delta(\overline{g},1)}\le s\le s_{\delta(1,n_{\overline{g}})}.\end{cases}$$

Similarly, we will use the notation $P'(s)$ to represent the derivative of $P(s)$. Since every two adjacent sections have one common section between them, to get a smooth curve, the height and slope of the beginning section of one segment should be equal to the height and slope of the ending section of the previous segment.

Remark 1. For a long road, the value of s in $P_g(s)$ could be large. Since $P_g(s)$ has a square term of s, numerical errors can happen for large values of s. To eliminate these numerical errors from the implementation of the models, each value of s is calculated with respect to the beginning of its segment, i.e., $s-s_{\delta(g,1)}\ \forall g\in\mathcal{G}$. Since this does not change the models, for the ease of describing the models, we use s instead of $s-s_{\delta(g,1)}$ in $P_g(s)$.

Side-slope refers to the gradual decrease and increase in height from the road profile to the ground profile of a cut and fill section respectively. Side-slopes are very important from the civil engineering point of view because they help to make the road durable. Therefore, to model the vertical alignment problem accurately,