Computers & Operations Research 44 (2014) 193–205

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

A dual bounding scheme for a territory design problem

Mónica G. Elizondo-Amaya [a,*], Roger Z. Ríos-Mercado [a], Juan A. Díaz [b]

[a] Graduate Program in Systems Engineering, Universidad Autónoma de Nuevo León, AP 111-F, Cd. Universitaria, San Nicolás de los Garza, Nuevo León 66450, Mexico
[b] Department of Industrial and Mechanical Engineering, Universidad de las Américas Puebla, Sta. Catarina Mártir, San Andrés Cholula, Puebla 72820, Mexico

ARTICLE INFO

Available online 16 November 2013

Keywords:
Commercial territory design
Discrete location
Lagrangian relaxation
Dual bounding scheme

ABSTRACT

In this work, we present a dual bounding scheme for a commercial territory design problem. This problem consists of finding a p-partition of a set of geographic units that minimizes a measure of territory dispersion, subject to multiple balance constraints. Dual bounds are obtained using binary search over a range of coverage distances. For each coverage distance a Lagrangian relaxation of a maximal covering model is used effectively. Empirical evidence shows that the bounding scheme provides tighter lower bounds than those obtained by the linear programming relaxation. To the best of our knowledge, this is the first study about dual bounds ever derived for a commercial territory design problem.

1. Introduction

Territory design can be viewed as the problem of grouping small geographical areas, called *basic areas*, into larger geographic clusters called *territories* according to specific planning criteria. These problems arise in different applications such as political districting [17,23,24,35,30,2,3] and sales territory design [43,45,46,14,9,22] to name the most relevant. An extensive survey on general territory design problems and their approaches can be found in Kalcsics et al. [26] and Duque et al. [10].

The problem addressed in this paper is motivated by a concrete practical application from a local beverage firm. To improve customer supply, the company needs to divide the set of city blocks (or basic units) in the city area into a specific number of disjoint territories. In particular, the planning requirements considered in this problem are territory compactness and territory balancing with respect to two activity measures present at every basic unit. The former criterion means that customers within a territory are relatively close to each other while the latter requirement refers to creating territories of about equal size in terms of both number of customers and product demand. This problem can be classified as a commercial territory design problem (TDP) for which related versions under different requirements have been addressed in literature from both exact and heuristic approaches.

Typically, the problem is modeled as minimizing a dispersion measure subject to some planning requirements such as connectivity and territory balancing. The connectivity requirement implies that basic units (BUs) that are assigned to the same territory must reach each other by traveling within the territory. Depending on how the dispersion measure objective is chosen, we can further classify these TDP models as p-median TDPs (PMTDP) and p-center TDPs (PCTDP). Heuristic methods have been developed for both different versions PCTDPs and PMTDPs. Ríos-Mercado and Fernández [36] introduced the PCTDP subject to connectivity and multiple balance constraints. They propose a Reactive GRASP to solve the problem. Their proposed approach obtained solutions of much better quality (in terms of dispersion measure and the balancing requirements) than those found by the company method in relatively fast computation times.

Later, Caballero-Hernández et al. [4] study other version of the commercial PCTDP model that includes additional joint assignment constraints which means that some units are required to belong to the same territory. In that work, the authors develop a metaheuristic solution approach based on GRASP. Experimental results show the effectiveness of their method in finding good-quality solutions for instances up to 500 BUs and 10 territories in reasonably short computation times. Particularly, a very good performance is observed within the local search procedure, which produces an improvement of about 90% in solution quality.

Ríos-Mercado and Salazar-Acosta [38] address an extension of the TDP that considers requirements about design and routing in territories. In contrast to the TDP variations described above, the authors use network-based distances between BUs (instead of Euclidean distances) and a diameter-based function to measure territory dispersion. To solve this problem, the authors proposed a GRASP that incorporates advanced features such as adaptive memory and strategic oscillation. Empirical evidence shows that the incorporation of these two components into the procedure had a very positive impact on both obtaining feasible solutions and improving solution quality.

* Corresponding author. Tel.: +52 18186877231.
E-mail addresses: moni@yalma.fime.uanl.mx (M.G. Elizondo-Amaya), roger.rios@uanl.edu.mx (R.Z. Ríos-Mercado), juana.diaz@udlap.mx (J.A. Díaz).

0305-0548/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cor.2013.11.006

Salazar-Aguilar et al. [39] present an exact optimization framework based on branch and bound and cut generation for tackling relatively small instances of several TDP models. Particularly, they studied both, the PCTDP and PMTDP models. They successfully solved instances of up to 100 BUs for the PCTDP and up to 150 BUs for the PMTDP. The authors also propose new integer quadratic programming models that allowed to efficiently solve larger instances by commercial MINLP solvers. For IQPs models, they obtained locally optimal solutions for instances with up to 500 BUs and 12 territories.

Ríos-Mercado and López-Pérez [37] and López-Pérez and Ríos-Mercado [28] address a commercial TDP with additional side constraints such as disjoint assignment requirements and similarity with existing plan. In their work, they assume a fixed set of centers, and present several heuristic algorithmic strategies for solving the allocation phase.

Recently, a bi-objective TDP model was introduced by Salazar-Aguilar et al. [40], where an $\varepsilon-$constraint method is developed for tackling small- to medium-scale instances from an exact optimization perspective. In that work, two different measures of dispersion are studied, one based on the p-center problem objective and the other based on the p-median objective model. It was shown how the latter had a tighter LP relaxation that allowed to solve larger instances. The proposed method was successful for finding optimal Pareto frontiers on instances from 60 up to 150 BUs and 6 territories. It was also clear that larger instances were indeed intractable, thus justifying the use of heuristic approaches proposed by Salazar-Aguilar et al. in [41,42]. In these works, the authors address the development of GRASP and Scatter Search (SS) strategies to handle considerably large instances. These proposed heuristic procedures outperformed two of the well-known and most successful multiobjective algorithms in the field, the Non-dominated Sorting Genetic Algorithm (NSGA-II) by Deb et al. [8] and the Scatter Tabu Search Procedure for Multiobjective Optimization (SSPMO) by Molina et al. [32].

As it can be seen, from literature, practically all of the work on commercial territory design has focused on developing heuristics for finding good feasible solutions to large instances in reasonable times due to the well established NP-completeness of both PCTDP and PMTDP [36,39]. However, thus far, the quality of the solutions obtained by these heuristic methods has not been properly assessed since the quality of the lower bound provided by the linear programming relaxation of TDP models is very poor. To the best of our knowledge, no dual bounding schemes have been developed for any of the commercial TDP models found in the literature. It is worth mentioning that besides being useful in evaluating the quality of heuristic solutions, dual bounds are also the foundations in the development of exact solution methods.

Therefore, the main contribution of this work is the introduction and development of the first dual bounding scheme for a commercial territory design problem. The TDP addressed here considers balance and compactness requirements. This scheme is motivated by exact solution methodologies already found in literature for related location problems, where the main idea is to generate and solve a set of auxiliary problems. Particularly, Albareda-Sambola et al. [1] propose a successful exact solution method for the capacitated p-center problem (CpCP) that involves a procedure for obtaining lower bounds for this problem. The bounding procedure developed in [1] is not quite applicable for our problem; however, given the strong similarities, one of the goals of this paper is to extend this bounding procedure to handle multiple balance constraints.

The proposed algorithm performs a binary search over a specific set of covering radii extracted from the distances matrix and solves for each of them a Lagrangian dual problem based on a maximal demand covering problem. The evaluation of this dual problem for a given radius δ can determine, under certain conditions, when such covering radius is a dual bound for TDP. An empirical study was carried out on a collection of data instances. The results show the effectiveness of the developed scheme as it considerably outperforms the linear programming relaxation dual bound.

The paper is structured as follows. Section 2 defines the problem formally and describes the mathematical formulation. Section 3 presents the dual bounding scheme and each of its components. Experimental work is included in Section 4. Finally, conclusions and some final remarks are drawn in Section 5.

2. Problem description

Let V be a set of nodes or BUs representing city blocks. Let w_i^a be the measure of activity a in block i, $a \in A = \{1,2\}$ where $a=1$ denotes number of customers and $a=2$ denotes product demand. Let d_{ij} be the Euclidean distance between each pair of basic units i and j. The number of territories is given by p. A territory design configuration is a p-partition of the set V. Let $w^a(V_k) = \sum_{i \in V_k} w_i^a$ be the size of territory $V_k \subseteq V$ with respect to activity a. A solution to this problem must have balanced territories with respect to each activity. Due to the discrete nature of the problem and to the unique assignment constraints, it is practically impossible to get perfectly balanced territories. Thus, in order to address this issue, a tolerance parameter τ^a for each activity a is introduced. This tolerance parameter is user specified and it represents a limit on the maximum deviation allowed from an ideal target. This target value is given by the average size $\mu^a = w^a(V)/p$. Finally, in each of the territories, basic units must be relatively close to each other. To account for this, in this work we use a dispersion function based on the p-center problem objective.

All parameters are assumed to be known with certainty. Therefore, the problem can be formally described as finding a p-partition of a set V of basic units that meets multiple balance constraints and minimizes a dispersion measure.

2.1. Integer programming formulation

To state the model mathematically, we define the following notation:

Indices and sets

V	set of BUs,
A	set of BUs activities,
i,j	BUs indices; $i,j \in V = \{1,2,\ldots,n\}$,
a	activity index; $a \in A = \{1,2\}$.

Parameters

n	number of BUs,
p	number of territories,
w_i^a	value of activity a in node i; $i \in V$, $a \in A$,
d_{ij}	Euclidean distance between i and j; $i,j \in V$,
τ^a	relative tolerance with respect to activity a; $a \in A$, $\tau^a \in [0,1]$.
μ^a	$w^a(V)/p$, average (target) value of activity a; $a \in A$.

Although the practical decision does not require to place facilities on centers as it is done in location problems, we used binary decision variables based on centers because they allowed to model territory dispersion appropriately.

Decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if BU } j \text{ is assigned to territory with center in BU } i, \\ 0 & \text{otherwise.} \end{cases}$$