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The multimode covering location problem

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ABSTRACT

In this paper we introduce the *Multimode Covering Location Problem*. This is a generalization of the Maximal Covering Location Problem that consists in locating a given number of facilities of different types with a limitation on the number of facilities sharing the same site.

The problem is challenging and intrinsically much harder than its basic version. Nevertheless, it admits a constant factor approximation guarantee, which can be achieved combining two greedy algorithms. To improve the greedy solutions, we have developed a *Variable Neighborhood Search* approach, based on an exponential-size neighborhood. This algorithm computes good quality solutions in short computational time. The viability of the approach here proposed is also corroborated by a comparison with a *Heuristic Concentration* algorithm, which is presently the most effective approach to solve large instances of the Maximal Covering Location Problem.

1. Introduction

A facility location problem consists in placing a number of facilities to serve a set of demand centers, whose positions are known, while optimizing a given objective function. The problem admits several variants, based on the objective of the decision maker and on the application setting. For a complete taxonomy of facility location problems, the interested reader may refer to ReVelle et al. [21].

In this paper we focus on a generalized version of the *Maximal Covering Location Problem* (*MCLP*), first proposed by Church and ReVelle [6]. The *MCLP* belongs to the class of discrete location problems, i.e., problems with a finite set of demand centers and a finite set of candidate locations. The *MCLP* does not require all the demand centers to be served: its purpose is to locate a given number of facilities maximizing the number, or the total weight, of the served demand centers. Because of its wide applicability in the real world, especially in the planning of service and emergency facilities, the *MCLP* is a well-studied problem. Chung [5] reviewed several other applications of the *MCLP*, such as data abstraction, stock selection and classification problems. Other interesting applications are those described by Dwyer and Evans [13] for the selection of mailing lists, Daskin et al. [11] for flexible manufacturing, Hougland and Stephens [18] for air pollution control and Melo et al. [19] for supply chain management.

Since its proposal, the *MCLP* has been generalized in different ways. Berman et al. [4] reviewed gradual cover models, cooperative cover models and variable radius models. Ghiani et al. [15] introduced a capacitated plant location problem where multiple facilities can be opened in the same site. Rajagopalan et al. [20] considered a multiperiod set covering location model in the field of application of emergency medical services. Dell'Olmo et al. [12] tackled the optimal location of intersection safety cameras on an urban traffic network to minimize the impact of car accidents, through a multiperiod variant of maximal covering location.

In this paper, we present the *Multimode Covering Location Problem* (*MM-CLP*). This problem consists in placing a given number of facilities of different types (hereafter called *modes*) to serve demand centers that require different types of service. The goal is to maximize the demand coverage over all the considered modes. An additional restriction with respect to the *MCLP* is that only a limited number of different modes can be activated in each candidate facility site. A similar generalization for the uncapacitated facility location problem has been recently proposed by Arora et al. [2]. They present a 4-approximation LP-rounding based algorithm for a class of problems with only two modes.

Possible applications of the *MM-CLP* refer to the distribution of facilities addressed to different users in the same area (e. g., fire stations and police stations). A similar situation occurs in the location of a heterogeneous fleet of ambulances, some of which might have an equipment and crew specialized in the treatment of heart-strokes or other severe health conditions. Another common application of covering location problems is in the design of nature reserves for the protection of biodiversity: each land parcel can be

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subject to different types of protection, with a different impact on the endangered species which populate the parcel. Finally, telecommunication antennas with different radiuses and pointing in different directions can be installed in the same site, covering different subsets of users, but the number of antennas in each site can be limited by the available space and by the need to avoid reciprocal interferences.

The *MM-CLP* is $\mathcal{NP}$-hard because it includes the *MCLP* as a special case. However, while medium-size instances of the *MCLP* can be quickly solved by commercial solvers, even fairly small instances of the *MM-CLP* prove much harder. Nevertheless, we here prove that the *MM-CLP* admits two greedy algorithms with a constant factor approximation guarantee. This extends in a non-trivial way a similar property of the *MCLP*. To compute better solutions, we also present a *Variable Neighborhood Search* (*VNS*) algorithm, which implements a *Very Large Scale Neighborhood Search* (*VLNS*) as its basic local search procedure. The hybridization of *VNS* with other metaheuristic approaches is an active field of research, see for example [9,17,3]. To evaluate the performance of the proposed *VNS* approach, we have first compared it to a simpler *VNS* implementation based on a polynomial-size neighbourhood. Then, we have implemented an alternative approach, based on the *Heuristic Concentration* (*HC*) framework developed by ReVelle et al. [22]. To the best of our knowledge, *HC* is considered the state-of-the-art heuristic for the *MCLP*.

The remainder of the paper is organized as follows. In Section 2 we formally define the problem, through a mathematical programming formulation. The complexity and approximation properties of the *MM-CLP* are described in detail in Section 2.1. In Section 3, we describe the *VNS* framework. Section 4 reports a computational comparison of the *VNS* and *HC* algorithms on a set of randomly generated benchmark instances, showing that the former clearly outperforms the latter. Finally, Section 5 draws some conclusions.

2. Mathematical model

Let I be a set of demand centers, J a set of candidate facility sites and M a set of modes. The relation between facility sites and demand centers in each mode can be represented with a binary matrix: $a_{ijm} = 1$ if facility site j is able to serve demand center i in mode m and $a_{ijm} = 0$ otherwise. For each candidate facility site $j \in J$, there is a maximum number b_j of modes that can be activated on the site. The number of facility sites used in each mode, K_m, is given and a weight w_{im} is assigned to each demand center $i \in I$ and mode $m \in M$. The *MM-CLP* requires to find a subset of facility sites for each mode, such that the total weight of the served demand centers is maximum.

Let $x_{jm} = 1$ if a facility of mode $m \in M$ is located on site $j \in J$, $x_{jm} = 0$ otherwise; $y_{im} = 1$ if demand center $i \in I$ is served in mode $m \in M$, $y_{im} = 0$ otherwise. The *MM-CLP* can be formulated as follows.

$$\max z = \sum_{i \in I} \sum_{m \in M} w_{im} y_{im} \tag{1a}$$

$$\sum_{j \in J} a_{ijm} x_{jm} \geq y_{im} \quad i \in I, m \in M \tag{1b}$$

$$\sum_{j \in J} x_{jm} = K_m \quad m \in M \tag{1c}$$

$$\sum_{m \in M} x_{jm} \leq b_j \quad j \in J \tag{1d}$$

$$x_{jm} \in \{0,1\} \quad j \in J, m \in M \tag{1e}$$

$$y_{im} \in \{0,1\} \quad i \in I, m \in M \tag{1f}$$

The objective function (1a) maximizes the total weight of the served demand centers. The covering constraints (1b) link the x and y variables. For each mode, Constraints (1c) fix the number of facilities to be placed. Constraints (1d) set the maximum number of facilities (of different modes) that can be located in each site. The integrality of the x and y variables is imposed by Constraints (1e) and (1f).

We here add a few remarks about the formulation. First, note that, once the x variables are fixed, the objective function and the covering constraints implicitly assign integer values to the y variables. Therefore, Constraints (1f) can be relaxed to $0 \leq y_{im} \leq 1$ for all $i \in I, m \in M$. Second, the feasibility of the problem depends only on the cardinality constraints (1c) and (1d): the problem is feasible if and only if $\sum_{m \in M} K_m \leq \sum_{j \in J} b_j$. From the practical point of view this condition is in general satisfied, because the number of candidate facility sites exceeds the number of facilities to be located. If the problem is feasible, Constraints (1c) can be relaxed to $\leq$ inequalities.

2.1. Complexity and approximation properties

The decision version of the *MM-CLP* is $\mathcal{NP}$-complete, because the special case in which the set of modes is a singleton ($|M| = 1$) coincides with the *MCLP*. An alternative reduction to the *MCLP* can be obtained allowing each candidate site to use all available modes ($b_j = |M|$ for all $j \in J$). Under this assumption, in fact, Constraints (1d) are redundant and the problem decomposes into $|M|$ independent instances of the *MCLP*, one for each mode.

The *MM-CLP* also includes as a special case the *k-MCLP*, which requires to compute k disjoint solution of the *MCLP* such that the sum of their values is maximum. This problem corresponds to instances of the *MM-CLP* in which the number of modes is fixed to k, only one facility can be located in each site ($b_j = 1$) and the facilities serve the same demand centers for all the modes.

In what follows, we present approximation properties for the *MM-CLP*. These properties generalize the approximation results provided by Vohra and Hall [24] for the *MCLP*. A maximization problem is α-approximable when it admits a polynomial time algorithm that provides on each instance P a solution $\tilde{z}(P)$ such that $\tilde{z}(P)/z^*(P) \geq \alpha$, where $z^*(P)$ is the value of the optimal solution of P [14].

With the purpose of establishing the approximation results, we first present two greedy algorithms. As it is customary for the *MCLP*, in what follows we will also denote the facility sites as *columns* and the demand centers as *rows*.

Algorithm $Greedy1(I, J, M, a, b, K, w)$
$x_{jm} := 0$ for all $j \in J$, $m \in M$;
$I_{jm} = \{i \in I : a_{ijm} = 1\}$ for all $j \in J$, $m \in M$;
for $m := 1$ to $|M|$ **do**
  for $j := 1$ to K_m **do**
    $j^* := \arg\max_{j \in J} \sum_{i \in I_{jm}} w_{im}$;
    $x_{j^*m} := 1$;
    $I_{jm} := I_{jm} \setminus I_{j^*m}$ for all $j \in J$;
    if $\sum_{m \in M} x_{j^*m} = b_{j^*}$ **then** $J := J \setminus \{j^*\}$;
  end for
end for
return x;

Fig. 1. Pseudocode of Algorithm *Greedy1*.