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A two-stage robust optimization approach for the mobile facility fleet sizing and routing problem under uncertainty

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ABSTRACT

We propose a two-stage robust optimization model for the mobile facility fleet sizing and routing problem with demand uncertainty. A two-level cutting plane based method is developed, which includes an algorithm to generate problem-specific lower bound inequalities in the outer level, and a hybrid algorithm in the inner level that combines heuristic and exact methods to solve the recourse problem. Numerical tests show that the design and operation from the proposed method outperforms other solution approaches. The efficiency of the proposed solution algorithm in identifying the optimal solution is quantified and the robustness of the proposed model is demonstrated for varying degrees of uncertainty in demand.

1. Introduction

The traveling salesman problem (TSP) and the vehicle routing problem (VRP), as classical operations research problems, have been studied extensively to handle the planning and scheduling of the vehicle movement within a network. Vehicles in those problems usually provide pickup or delivery services between facilities (e.g., depots or distribution centers) and customers (e.g., retailers or end users). However, in reality, vehicles themselves can sometimes act like a facility to provide real-time services to customers when they are stationary at one location. More importantly, they have the advantage of moving from one place to another like ordinary vehicles to cover a broader region, a feature especially important for regions that exhibits a large variability in demand over time. For instance, light trucks equipped with cellular base stations provide cellular services to areas where the established cellular network is temporarily insufficient [28]. Also, vans provide fast food services to customers in different regions in different time periods of the day [32]. Note that when these vehicles are in service, they behave like traditional facilities, and the duration that the vehicle stays at a customer point is directly related to the amount of service provided. In this paper, we use the term mobile facility (MF) to denote this “facility-like-vehicle”. The MFs considered here share the same features as those shown in Lei et al. [32]: (1) the demand served at a location is proportional to the duration of time an MF remains stationary at that location, (2) the MFs cannot provide services when they are in motion, and (3) the service begins when an MF arrives at a location and ends when it departs.

We illustrate these features through a simple example as shown in Fig. 1. There are three customer points (C1, C2 and C3) and three candidate locations (L1, L2 and L3) where the MF can stay. We consider a 2-h planning horizon (from 8:00AM to 10:00AM) with eight time periods (i.e., 15-min periods). Each customer is covered by a single location as shown in Fig. 1. The travel time between any two locations is assumed to be 15 min. The amount of demand in each time period is given in Table 1.

In Fig. 2, we show the service schedule of a single MF. The horizontal arrows indicate that the MF is stationary at a certain location and is available to provide service. As shown in the figure, the MF stays at L1 for two time periods from 8:00, covering 20 units of demand from C1. It then departs from L1 at 8:30 and arrives at L3 15 min later. When the MF is on the road, it is unable to provide services, and the amount served from 8:30 to 8:45 is zero. The MF then proceeds to L3 and finally ends its services at L2 as shown in the figure. We can see that the MF will travel to another location only if the gain from covering more demands at a different location outweighs the loss resulting from the movement.

Fleet sizing is one of the most important decisions as it is a major fixed investment for starting any business. Moreover, fleet sizing is a complex long-term decision which needs to be considered in conjunction with the operation level.

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First, it is essential to come up with certain strategies to handle the uncertainty in demand in operation when making long-term investment decisions. In this paper, we focus on the robust optimization (RO) approach of modeling the proposed problem under demand uncertainty. The approach is appealing in three aspects: (1) it generates a solution immune to uncertain demand within a deterministic uncertainty set. The RO approach is quite desirable since the establishment of a fleet typically involves a significant investment in the starting stage of the business, including the cost of purchasing an MF, expenses for devices to be equipped on an MF for providing relevant service, the cost of the crew who operates the equipment, the routine maintenance cost, etc.; (2) the RO approach requires only moderate information about the uncertain demand data, rather than a detailed description of the probability distribution function or a large collection of sample scenarios; (3) the RO approach generates solutions with different level of conservativeness by manipulating the "budget of uncertainty" to guard against overly conservative results. The managers thus can be provided with more alternatives so that they can decide based on their specific needs. In this paper, the uncertain parameters are assumed to be within a polyhedral uncertainty set in line with those assumed in Bertsimas and Sim [11,12].

Second, the fleet size is strongly dependent on the effectiveness in fleet (such as the route and schedule planning). Moreover, owing to the "facility-like-vehicle" feature of the MF, the allocation of demands to stationary MFs is significant for the entire system performance. Hence, it is more reasonable to integrate the strategic level fleet sizing problem, the tactical level routing and scheduling problem, and the operational level allocation problem together into the decision making process. We name this type of problem as the *mobile facility fleet sizing and routing problem* (MFFSRP).

This paper considers the MFFSRP by applying a two-stage robust optimization modeling approach, in which the customer demand is uncertain with no information about the underlying probability distribution function. The idea of the two-stage RO approach is that the first-stage decisions (i.e., fleet sizing and routing plan) are made before the realization of uncertainty, and then the second-stage decisions (i.e., allocation of demands to the MFs) are made as recourse actions after the uncertainty has been known. The objective is to optimize the performance of the system over the entire planning horizon.

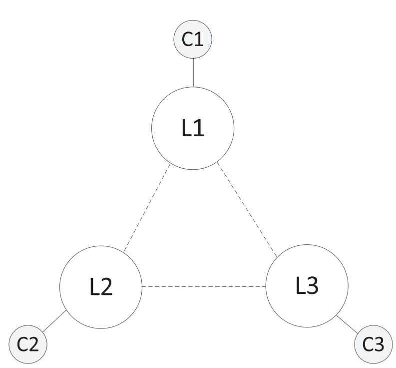

Fig. 1. Schematic representation of customers and candidate locations.

The computational aspect of the proposed problem must be carefully addressed. In addition to the integration of the fleet sizing and the routing problem, some other factors, such as the two-stage feature, uncertainty in demand, and multiple time periods of planning, all increase the computational complexity significantly. In this paper, we develop some new techniques to improve the performance of the conventional cutting plane method.

To the best of our knowledge, this study is the first one applying the RO approach in the context of the mobile facility planning field. Specifically, the contributions of this paper are as follows:

1. A two-stage RO approach is applied to the formulation of the MFFSRP for the first time, in which the uncertainty in demand is modeled as a polyhedral set to allow flexible control of the level of conservativeness. Numerical results reveal that the proposed RO approach has some distinctive advantages in optimizing the performance of the system compared to the overly conservative solution obtained from the box uncertainty set model.
2. An enhanced version of the cutting plane method is proposed. The enhancement includes: (1) substituting the single optimality cut by multiple cuts, (2) constructing several problem specific lower bound inequalities to tighten the lower bound of the master problem, and (3) developing a hybrid algorithm solver for the recourse problem.
3. An experiment is designed to examine the effectiveness of the proposed approach. The experiment goes beyond the commonly used measures for testing solution quality and computation time. Instead, we consider a wide class of performance measures to test the model performance, including economic effectiveness, reliability, robustness against variations in demand, solution efficiency, etc.

The remainder of the paper is organized as follows. In Section 2, we review some of the relevant literature with respect to the fleet sizing problem, the mobile facility planning problem, and the robust optimization approach. Section 3 first gives the mathematical formulation for the nominal model of the MFFSRP and then the two-stage robust formulation with the polyhedral uncertainty set. A two-level cutting plane type algorithm is presented in Section 4. Extensive numerical experiments are carried out in Section 5, and insights from the results are presented. This is followed by Section 6 with conclusions summarizing the main features of the proposed approach.

2. Literature review

In this section, we review the relevant literatures about the MFFSRP and the RO approach, especially the study of the mobile facility itself, the two-stage RO modeling, and the corresponding solution methodology.

It should be noted that the concept of mobile facilities is not new. Bespamyatnikh et al. [15] and Durocher and Kirkpatrick [22] studied a problem related to mobile facility location, where both

Table 1
Customer demand in different time periods.

Customer	Time							
	8:00–8:15	8:15–8:30	8:30–8:45	8:45–9:00	9:00–9:15	9:15–9:30	9:30–9:45	9:45–10:00
C1	10	10	10	0	0	0	0	0
C2	0	0	0	0	0	0	20	20
C3	0	0	0	15	15	15	0	0