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Adaptive differential evolution algorithm with novel mutation strategies in multiple sub-populations

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ABSTRACT

Differential evolution (DE) algorithm has been shown to be a very effective and efficient approach for solving global numerical optimization problems, which attracts a great attention of scientific researchers. Generally, most of DE algorithms only evolve one population by using certain kind of DE operators. However, as observed in nature, the working efficiency can be improved by using the concept of work specialization, in which the entire group should be divided into several sub-groups that are responsible for different tasks according to their capabilities. Inspired by this phenomenon, a novel adaptive multiple sub-populations based DE algorithm is designed in this paper, named MPADE, in which the parent population is split into three sub-populations based on the fitness values and then three novel DE strategies are respectively performed to take on the responsibility for either exploitation or exploration. Furthermore, a simple yet effective adaptive approach is designed for parameter adjustment in the three DE strategies and a replacement strategy is put forward to fully exploit the useful information from the trial vectors and target vectors, which enhance the optimization performance. In order to validate the effectiveness of MPADE, it is tested on 55 benchmark functions and 15 real world problems. When compared with other DE variants, MPADE performs better in most of benchmark problems and real-world problems. Moreover, the impacts of the MPADE components and their parameter sensitivity are also analyzed experimentally.

1. Introduction

With the development of science and technology, there are more and more global optimization problems arising in almost every application field. Generally, a global numerical optimization problem can be defined as follows.

$$\text{Minimize } f(X), X=(x_1, x_2, \ldots, x_D) \in S \qquad (1)$$

where $S \subseteq \prod_{i=1}^{D}[x_{i,L}, x_{i,U}]$, $-\infty < x_{i,L} < x_{i,U} < +\infty$ for all $i = 1, 2, \ldots, D$, and the symbol D denotes the dimensions of the decision variables. The target of Eq. (1) is to find a solution $X^* \in S$, which should satisfy the condition that $f(X^*) \le f(X)$ for $\forall X \in S$. Over the last two decades, global optimization problems have attracted a great interest of researchers and numbers of nature-inspired intelligent algorithms have been accordingly proposed for solving global optimization problems, such as genetic algorithm [1], memetic algorithm [2], differential evolution (DE) [3–8], particle swarm optimization [9] and artificial immune algorithm [10]. Among them, DE algorithm is a simple yet effective heuristic algorithm firstly proposed by Storn and Price [3] for dealing with global optimization over continuous space. Due to its outstanding characteristics, such as compact structure, ease to use, speediness and robustness, it has become more and more popular and been extended to handle a variety of optimization problems including multimodal [11], constrained [12], large-scale [13], multi-objective [14], dynamic optimization problems [15] and numbers of real-world applications [16–19].

As a new branch of evolutionary algorithm (EA), DE algorithm shares a similar structure with EA, which includes three important evolutionary operators, *i.e.*, mutation, crossover and selection. The performance of DE primarily relies on these evolutionary operators and their associated parameter settings, such as the scaling factor F and crossover rate Cr. Especially for mutation operator, it generates the mutant vectors to explore the search space and mainly affects the evolutionary direction. Numbers of DE mutation strategies have been proposed and investigated in detail, such as DE/rand/1, DE/rand/2, DE/best/1, DE/best/2, DE/current-to-best/1, DE/current-to-rand/1 [3,20,21] and other variants of them [4–8]. It is experimentally found that different DE mutation strategies have distinct characteristics, which may behave pretty differently in solving various kinds of global optimization problems [22]. For example, when solving unimodal problems, DE/best/1, DE/best/2,

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DE/rand-to-best/1 and DE/current-to-best/1 are demonstrated to have a fast convergence speed and very good performance, all of which employ the best solution found so far to do further exploration. However, they may lead to premature convergence easily when tackling multimodal problems. On the contrary, DE/rand/1 and DE/rand/2 have a slow convergence speed but strong exploration capability to prevent premature [4]. DE/current-to-rand/1 is a rotation-invariant strategy, which is more suitable for solving the rotated problems than the other DE strategies [21]. On the other hand, different parameter settings also have great impacts on their performances when solving various global optimization problems or even the same problem at different evolutionary stages. For example, a small crossover rate *Cr* in DE is generally suitable for separable functions while a large one is effective for non-separable functions [23]. Generally speaking, a large scaling factor *F* in DE is required at the early stage of the evolution to preserve the population diversity, while a small *F* is preferred to accelerate the population convergence at the later stage. In other words, no any specific parameter setting can always perform very well for all types of global optimization problems [24]. Therefore, in order to effectively use DE for specific optimization problem, a DE strategy is firstly determined and then its corresponding parameter settings are obtained by using a trial-and-error procedure. However, this procedure is very time-consuming and may not be practicable sometime. To overcome this inconvenience, a variety of effective approaches have been proposed to analyze the effects of DE strategies and their associated parameter settings, including the design of new DE strategies [24–29], the adaptive adjustment of multiple DE strategies and their parameter settings [4–7,20], and the combination of multiple DE strategies and parameter settings [8,30,31]. The detailed introductions of these relevant approaches are given in Section 3.

However, when tackling the vast complicated problems in practical applications, in our opinion, the optimization capability of DE is still insufficient as the performance of DE may deteriorate quickly with the increase of dimensionality in search space. This is mainly because it may easily fall into local optimum causing premature convergence or stagnation, especially for solving multimodal problems [32]. Inspired by the working specialization in practical work that different employers with special capabilities are only responsible for certain tasks, this paper proposes a novel adaptive DE algorithm using multiple sub-populations (MPADE) to further enhance the optimization performance. The parent population is divided into three sub-populations based on the individuals' fitness and then each sub-population takes on its search task for either exploitation or exploration. Moreover, an adaptive parameter control strategy is designed for each sub-population to adjust the parameter settings in an online manner and a simple replacement strategy is presented to adequately exploit the useful information from the trial vectors and target vectors. By this way, our proposed algorithm balances very well between exploration and exploitation to enhance the algorithmic robustness and optimization performance. Compared with the existing DE strategies [4–8,31], the contributions of this paper can be summarized as follows.

(1) Based on the individuals' fitness, the parent population is split into three sub-populations as respectively denoted by the inferior, medium, and superior sub-populations. They are evolved using different DE strategies to take on the responsibility of either exploration or exploitation, which is distinctly different from the existing DE strategies. As the inferior one includes the worse individuals that may be far away from the global optimal, DE/current-to-*r*best/2 mutation strategy is correspondingly performed to make a large perturbation on the inferior individual, which undertakes the exploration task. Regarding to the superior sub-population, DE/current-to-*n*best/2 is executed to disturb the better individual by a tiny perturbation, which launches a local search strategy and makes more exploitation of the current area. At last, DE/current-to-*p*best/2 is run on the medium sub-population to achieve a relative balanced strategy between exploration and exploitation.
(2) A simple yet effective adaptive scheme is designed to tune the parameter settings in multiple DE strategies. The corresponding scaling factor *F* and crossover rate *Cr* are dynamically adjusted based on the Cauchy and Gaussian random numbers at each generation. The control parameters of Cauchy and Gaussian distribution are updated based on the successful experience from the former evolutions.
(3) When selecting individuals to survive in the next generation, a replacement strategy is designed, which employs a few best individuals of trial population (offspring population) to replace the same number of the worst individuals in target population (parent population). It is only activated by a small probability that is gradually increased with the generation times. By this way, it can effectively accelerate the convergence speed at the later stage while not losing the population diversity at the beginning.

The advantages of MPADE are also validated by the experimental studies. In order to evaluate the comprehensive performance of MPADE, lots of experiments have been conducted on 55 benchmark problems from CEC2005 [39] and CEC2014 [43] competition on real parameter optimization, which includes various kinds of global optimization problems such as unimodal, multimodal, and complex hybrid composition functions, and 15 real word problems from CEC2011 [40]. When compared with various state-of-the-art DE algorithms (*i.e.*, CoDE [8], EPSDE [31], SaDE [4], jDE [5] and JADE [6], SAMODE [41]), and some recently proposed DE variants (*i.e.*, AEPD-JADE [44], DE-VNS [11], SinDE [42] and rank-jDE [27]), simulation results demonstrate that MPADE performs better on most of benchmark problems and some real world problems. At last, the influences of MPADE components and their parameter sensitivity are also studied experimentally.

The remainder of this paper is organized as follows. Section 2 introduces the basic DE algorithm, while Section 3 gives a review on the relevant research works. The details of our proposed MPADE algorithm are described in Section 4, which includes the sub-population division, the DE mutation strategies, the parameter adaptation scheme, and the replacement strategy. Experimental results of MPADE are presented in Section 5 and compared to various DE variants. Moreover, the effectiveness of MPADE and its parameters sensitivity are analyzed here. At last, Section 6 summarizes the conclusions and future work.

2. Basic differential evolution algorithm

Differential evolution is a branch of EA that follows the general procedures of EA. More specifically, there are three basic operators of DE, including mutation, crossover and selection, as described in the following subsections. In order to describe simply and clearly, some necessary notations and terminologies are introduced at first and then the basic operators of DE algorithm are described.

2.1. Notations and terminologies

Assume that a population with *Np* individuals is denoted by *P* and each individual is represented by the vector $X_i = (x_{1,i}, x_{2,i}, \ldots, x_{D,i})$, where *D* is the number of dimensions in solution space. Since the