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# Path based algorithms for metro network design

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ABSTRACT

This paper proposes a practical methodology for the problem of designing a metro configuration under two criteria: population coverage and construction cost. It is assumed that a set of corridors defining a rough a priori geometric configuration is provided by the planners. The proposed algorithm consists of fine tuning the location of single alignments within each corridor. This is achieved by means of a bicriteria methodology that generates sets of non-dominated paths. These alignments are then combined to form a metro network by solving a bicriteria integer linear program. Extensive computational experiments confirm the efficiency of the proposed methodology.



## 1. Introduction

In several cities, metro systems provide a desirable alternative to private transportation. They reduce traffic congestion, improve mobility and contribute to the protection of the environment. Designing a metro network is a complex problem in which several criteria must be considered, technical difficulties are important and the costs are very high. These problems also involve many different decision makers such as city planners, engineers, interest groups and politicians. For a recent overview of the metro design literature, see [11].

Because we are dealing with mobility issues, one of the goals when designing a metro system is to serve as many people as possible. It has been shown that people are willing to walk to a metro station provided that the walking distance to the metro station is not too large. Vuchic [24] states that this distance should not exceed 400 m (5 min). In metro location, two objective functions are typically considered: maximizing population coverage [5,8,16,18–20] and maximizing origin–destination trip coverage [10,15]. In most papers, construction cost is considered as a constraint. Here we jointly maximize population coverage and minimize cost in a bicriteria function. Typical constraints for the location of a metro line are the distance between stations (either consecutive stations in a line or between any two of them) and the total number of stations to be built.

The planners often have a rough idea about the general shape of the metro system to be constructed and about the major destinations it should serve, such as universities, hospitals, sport facilities, commercial centers and tourist attractions. Therefore, rather than looking at a representation of the whole city, broad corridors are defined within which the metro lines should be located. Several studies on real metro system network topologies [12–14] indicate that certain configurations are more effective than others. Thus Laporte et al. [12] have compared a number of measures on various generic metro configurations (see Fig. 1): (1) network complexity (number of edges divided by number of nodes); (2) connectivity (ratio of the number of edges compared to the maximal number of edges that could exist in a planar graph for a given number of nodes); (3) directness (proportion of origin–destination paths that can be traveled without transfers); and (4) passenger/network effectiveness (ratio of travel time with transfers to travel time disregarding transfers). These measures indicate that star configurations are the least complex, but far worse than triangle and cartwheel configurations in terms of connectivity, directness and passenger/network effectiveness. With respect to the same criteria, triangle configurations are slightly worse than cartwheels. In this paper we will restrict our attention to these three basic configurations. While stars are relatively inefficient, they are often preferred as initial configurations for new systems (e.g., Montreal and Minsk). With time these systems evolve and triangles are often the next step (e.g., the Minsk metro extension) and are used by some cities (e.g., Prague). Cartwheel-like configurations are common in some complex and mature metro systems such as those found in London or Moscow.

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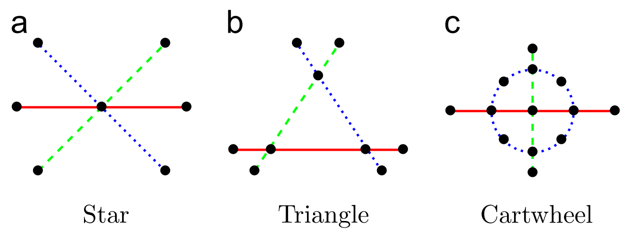


**Fig. 1.** Metro configurations.

In this paper we work with the modular approach first suggested by Bruno and Laporte [2]. The idea consists of first setting broad corridors whose combination defines a desired configuration such as a star, a triangle or a cartwheel. (In Bruno and Laporte [2] an interactive system allows planners to make a selection among a menu of predefined configurations and to define their own.) The system constructs an alignment within a given corridor, that is, a metro line defined by the location and sequence of its stations. This alignment minimizes a single cost. Because we consider two criteria, several non-dominated alignments are constructed within each corridor. The final metro configuration is then obtained by selecting one alignment per corridor through the solution of an integer linear program. With respect to Bruno and Laporte, we consider two criteria instead of one, and we generate several alignments for each corridor instead of only one.

The scientific contribution of this paper is algorithmic. We propose a modular heuristic capable of constructing a complex metro network by decomposing it into components and solving each component separately, thus considerably reducing the complexity of the construction process. The remainder of this paper is organized as follows. In the next section we present a mathematical model for the problem of locating a metro line and its stations. Section 3 is dedicated to the development of algorithms for the location of a metro line within a corridor, considering one or two criteria. In Section 4 integer formulations are introduced in order to optimally combine solutions of the problem in Section 3 according to classical metro system configurations with several corridors. In Section 5 we present and discuss results of extensive computational experiments. Conclusions are drawn in Section 6.

## 2. Mathematical model for the location of a metro line

We first present a model for locating a metro line in a general network representing a city, or part of it. We then explain how to obtain a network that represents a corridor within a city.

We consider an area in which a metro line is to be located as a network $G=(\mathcal{N},\mathcal{E})$, where $\mathcal{N}$ is the set of nodes, the possible station locations, and $\mathcal{E}$ is the set of edges, the links between stations. A metro line is represented by a sequence of stations connected by edges. A metro line corresponds to an undirected path of the form $p=\langle v_1,v_2,\ldots,v_\ell\rangle$, where $v_i\in\mathcal{N}$ for $i=1,\ldots,\ell$. For simplicity, we write $v_i\in p$ if the node $v_i$ appears on path $p$. Each node $i$ of the network is associated with the catchment area population of a station located there, $d_{ij}$. With each arc $(i,j)$ we associate with cost $c_{ij}$ of constructing a link between stations $i$ and $j$, and the Euclidian distance $l_{ij}$ between the nodes $i$ and $j$.

Given an initial and a terminal nodes, respectively $s$ and $t$, that correspond to ending stations, the problem of locating a metro line linking $s$ to $t$ is defined as

$$\text{maximize} \quad d(p)=\sum_{i\in p} d_i \tag{1}$$

$$\text{minimize} \quad c(p)=\sum_{(i,j)\in p} c_{ij} \tag{2}$$

$$\text{subject to} \quad \underline{l}(p)=\min\{l_{ij}:(i,j)\in p\}\geq m \tag{3}$$

$$\bar{l}(p)=\max\{l_{ij}:(i,j)\in p\}\leq M \tag{4}$$

$$h(p)\leq W \tag{5}$$

$$l_{ij}\geq D,\quad \forall i,j\in p \tag{6}$$

$$p \text{ a path between } s \text{ and } t. \tag{7}$$

Possible objective functions for this problem are $d$, providing a measure for the population coverage along the line associated with that path, and $c$, a measure for the construction cost along the same metro line. In terms of a metro line, the functions $\underline{l}$ and $\bar{l}$ represent the minimum and the maximum distances between two consecutive stations. Our goal is to locate a metro line that maximizes the population coverage defined by (1) (in the next section we will also include as second objective the minimization of the construction cost defined by (2)) over the set of all feasible paths.

Constraint (3) sets a lower bound on the distance between consecutive stops, with $m$ a given constant. In addition, for accessibility reasons two consecutive stations on a line should not be too far from each other, which is represented by (4), with $M$ another constant. Typical values for $m$ and $M$ are around 1 km and 2 km, respectively [24]. We also consider an upper bound on the number of stops allowed in a metro line, given by (5), with $h$ a function that assigns to each path its number of nodes and $W$ a predefined upper bound. This constraint is necessary since it is difficult to operate a metro line with too many stations.