Computers & Operations Research 62 (2015) 169–176

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

An integrated model for site selection and space determination of warehouses

Simin Huang [a], Qian Wang [b], Rajan Batta [c,*], Rakesh Nagi [d]

[a] Department of Industrial Engineering, Tsinghua University, Beijing 100084, PR China
[b] Management School, University of Chinese Academy of Sciences, Beijing 100190, PR China
[c] Department of Industrial and Systems Engineering, University at Buffalo (SUNY), Buffalo, NY 14260, USA
[d] Department of Industrial and Enterprise Systems Engineering, University of Illinois, Urbana, IL 61801, USA

ARTICLE INFO

Available online 5 November 2014

Keywords:
Warehouse
Location
Sizing
Concave minimization

ABSTRACT

In this paper we present an integrated model for site selection and space determination for warehouses in a two-stage network in which products are shipped from part suppliers to warehouses, where they are stored for an uncertain length of time and then delivered to assembly plants. The objective is to minimize the total inbound and outbound transportation costs and the total warehouse operation costs, which include the fixed costs related to their locations and the variable costs related to their space requirements for given service levels. Each warehouse is modeled as an $M/G/c$ queueing system in which each storage slot acts as a server. We formulate this problem as a nonlinear mixed integer program with a probabilistic constraint. Two cases are considered. For the continuous unbounded size case, we find an approximate formula for the overflow probability and reformulate the problem into a set-covering problem. For the discrete size option case, we reformulate the problem into a capacitated connection location problem with discrete size options. Computational experiments are performed and the results show that the continuous model is appropriate for the small and median size problems and the discrete model is a good choice for the large size problems.

1. Introduction

The logistic system studied in this paper can be depicted in Fig. 1. In such a system, parts are transported from suppliers to warehouses and then delivered to assembly plants. Since the assembly plants usually follow a make-to-order policy, the demand of parts is random, drawn from the warehouses by the plants. Due to the distance and other factors, the arrival of parts at the warehouse is also random. Hence, the storage levels at the warehouses are random too. Therefore, in some situations, the warehouse space constructed may be insufficient and an arriving product may find no storage slot available. Excess amount of space required has to be leased from a public warehouse, which is much more expensive. This logistic setting can be observed in the parts supply chain in some diesel engine manufacturing companies in China. The design of such a logistic network involves decisions on (i) the number of warehouses to open and the selection of their locations; (ii) the size of each open warehouse to satisfy a predetermined service level, where the service level is measured by the probability that an arriving product finds no storage slot available at a warehouse; (iii) the product flow assignment, i.e., product routing through supplier–warehouse–plant. The objective of the problem is to minimize the total inbound and outbound transportation costs and the total warehouse operation costs which include fixed costs related to their locations and variable costs related to their space requirements when certain warehouse service quality is guaranteed under stochastic product demand. This problem is an integration of the *warehouse location and allocation problem* and the *warehouse sizing problem*.

The warehouse location and allocation problem has received considerable attention in the past few decades. It focuses on the determination of the number of warehouses and their locations to open, as well as the product flow assignments in order to minimize the transportation cost and the fixed location cost. Reviews on the facility location problems with simplified static and deterministic models can be found in Krarup and Pruzan [9] and Sridharan [22]. Reviews for facility location under uncertainty can be referred from Snyder [21]. Reviews on the facility location problems with dynamic and stochastic models can be found in Owen and Daskin [13]. Reviews on hierarchical facility location models can be seen in Sahin and Sural [19]. Among the location theory literatures, the ones that are closely related to ours are the joint location-inventory models. Nozick and Turnquist [12] incorporated safety stock cost, service responsiveness and uncertainty

* Corresponding author.
E-mail address: batta@buffalo.edu (R. Batta).

http://dx.doi.org/10.1016/j.cor.2014.10.015
0305-0548/

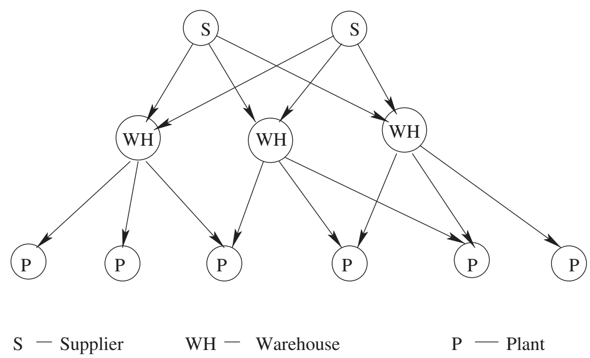

Fig. 1. A typical two-stage distribution system.

in delivery time into the traditional location models and approximated the safety stock cost at each distribution center by a linear regression function of the number of distribution centers. Shen et al. [20] studied a distribution network in which some of the retailers are allowed to serve as distribution centers to achieve risk-pooling benefits. The problem is to determine which retailers to serve as distribution centers and how to allocate the other retailers to the distribution centers. They reformulated the nonlinear integer-programming model as a set-covering model and proposed a column generation algorithm to solve the problem. Miranda and Garrido [11] presented a capacitated integrated location-inventory model and developed a Lagrangian relaxation algorithm to solve it. Teo and Shu [25] introduced a joint location-inventory model that considers multiple echelons inventory cost and proposed a column generation algorithm to solve the model. Ozsen et al. [14] extended Shen et al. [20] to the situation in which the costs at each warehouse exhibit initially economies of scale and then an exponential increase due to the capacity limitations. Benjaafar et al. [2] considered the problem of joint demand allocation and inventory control in a system with continuous time, multiple demand sources, multiple inventory locations, and a capacitated production system. Tsao et al. [26] developed a continuous approximation approach for the integrated facility-inventory allocation problem. However, in this class of problems, space requirements are not explicitly considered. Capacitated versions of the problem have been analyzed, which implicitly take into account of the space restrictions. Location of supplies in a disaster setting also has a relationship to our work, where the supplier corresponds to the central warehouse, the warehouse corresponds to supply depots, and the plant corresponds to demand points that result from the disaster. An excellent example of this type of model is the paper by Verma and Gaukler [27], who present a 2-stage program for locating disaster response facilities. They establish a direct connection between available capacity and proximity of optimal locations to high-risk areas.

The warehouse sizing problem has also received considerable attention in the past few decades. It seeks to find a warehouse size which either allows a required service level to be attained or minimizes the total costs. Under the assumption of constant product demand, Cormier and Gunn [3,4] coordinated the warehouse size with the inventory policy. Sung and Han [23] formulated the problem of determining the optimal size of an automated storage and retrieval system as queueing models. Multi-period warehouse leasing problems were investigated by White and Francis [28], Lowe et al. [10] and Rao and Rao [16]. The demand for the warehouse space is estimated by a discrete probability distribution. Jucker et al. [7] considered a multi-location warehouse leasing problem with uncertain demand. Roll and Rosenblatt [17] and Rosenblatt and Roll [18] developed simulation models to measure the relationship among warehouse size, inventory policy, and other various parameters. We note, however, that all of these models assume that locations have been selected.

Over the past decades there has been recognition of the fact that simultaneous consideration of warehouse location/routing and sizing yields better results than considering these problems separately. The first paper to recognize this was the work by Talavera [24] who did this in the context of a simulation study. This empirical finding due to simulation led other researchers to explore mathematical programming approaches to this simultaneous decision making context. Two examples of this work are that by Shen et al. [20] and that by Huang et al. [6].

Shen et al. [20] use demand flows from a single source to multiple destinations. Furthermore, they model the warehouse cost using an economic order quantity model with safety stock. Also, they focus on inventory cost reduction as the principal driving objective. Nevertheless, their work clearly illustrates the benefit of simultaneous decision making in warehouse location/routing and sizing. It provides an important building block for the work that we present in this paper. Our model generalizes the work of Shen et al. to the setting of multiple sources, instead of single source. We select a queueing model to mimic a warehouse, as opposed to an economic order quantity model. Also, we consider warehouse construct cost as the driving objective, as opposed to inventory cost. In terms of methodology, we lean on key results from Shen et al. to provide optimality conditions for our model. Essentially, we are able to demonstrate similar results in a different modeling context, while utilizing some of the key theoretical findings in their solution approach.

Huang et al. [6] consider simultaneous siting and sizing of distribution centers on a plane and present an analytical approach to solve the integrated decision making problem. Like the Shen et al. [20] paper, the work by Huang et al. [6] also provides a building block for the work that we present in this paper. We analyze the problem in a discrete choice setting for potential warehouse locations, in contrast to the planar setting of the Huang et al. paper. We model the warehouse as a queueing system which leads to a nonlinear mixed integer program with a probabilistic constraint, in contrast to the direct sizing cost approach that is used in the Huang et al. paper. We consider stochastic demands and handling times, in contrast to the deterministic distribution system assumption made in the Huang et al. paper. We further allow both continuous capacity and discrete capacity considerations, in contrast to the discrete capacity choice offered in the Huang et al. paper. Methodologically, the two papers take different approaches at solving their respective models. Essentially, the modeling contexts are significantly different and the results are similar in the sense of the benefit yielded from simultaneous consideration of warehouse location/routing and sizing.

The rest of the paper is organized as follows. Section 2 presents a mathematical formulation of the integrated warehouse site selection and space requirement problem. The continuous unbounded size case is studied in Section 3 and a column generation approach is proposed to solve the problem based on several established properties. The discrete size option case is analyzed in Section 4. Section 5 reports the computational results and provides a comparison of the two cases. Section 6 contains a summary and suggests the directions for future work.

2. Problem formulation

The problem considered in this paper can be described as follows. Products are shipped from a set of part suppliers to a set of assembly plants through a set of warehouses. The size of each warehouse is measured by the total number of storage slots (each assumed to be of equal size). If an arriving product finds that all