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The Weber problem in congested regions with entry and exit points

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ABSTRACT

The Weber problem is about finding a facility location on a plane such that the total weighted distance to a set of given demand points is minimized. The facility location and access routes to the facility can be restricted if the Weber problem contains congested regions, some arbitrary shaped polygonal areas on the plane, where location of a facility is forbidden and traveling is allowed at an additional fixed cost. Traveling through congested regions may also be limited to certain entry and exit points (or gates). It is shown that the restricted Weber problem is non-convex and nonlinear under Euclidean distance metric which justifies using heuristic approaches. We develop an evolutionary algorithm modified with variable neighborhood search to solve the problem. The algorithm is applied on test instances derived from the literature and the computational results are presented.

1. Introduction

In the classical Weber problem, a single facility is located to minimize the total weighted distance to a set of demand points. The real-life location problems, however, contain restrictions on the facility location or access routes that have to be considered in connecting facility to demand points. There might be some areas where facility cannot be located and traveling is restricted. If such limitations are incorporated, the problem is called the *restricted facility location problem* (RLocP) [1]. Three types of restricted regions are studied in the literature. If passing through the region is not allowed, it is called a *barrier* region (for instance, mountains and lakes that trouble traveling with cars) [2]. In *congested regions*, traveling is possible anywhere in the region but at an additional cost [3]. Urban zones and parks are examples of such regions. The other type of restricted areas is *forbidden regions* where the facility location is forbidden while traveling across the whole region is possible without any charge (e.g. protected areas where facilities cannot be located due to their environmental or cultural values) [4].

Refs. [4,5,6] showed how the restricted areas make the objective function and feasible region of the RLocP non-convex and discontinued, which make the problem more difficult to solve than the Weber problem. Refs. [1,7] provided an overview on the RLocP with various restriction types. The RLocP with barriers is comprehensively studied by [8]. There are limited number of studies in the literature about the RLocP with congested regions [3,9,10] which all consider the rectilinear distance metric and per-unit distance traveling penalty costs. Refs. [2,11] considered rectilinear distance norm and provided solution approaches based on the special grid structure of the RLocP with barriers and with forbidden regions instances.

Ref. [12] considered the RLocP with line barriers and considered a variant to the problem where there are a finite number of passages on the barrier. The authors studied the special characteristics of line barriers in the problem and provided a solution approach to reduce the problem into a polynomial number of unrestricted convex problems. Refs. [6,13] considered Euclidean distances in the RLocP with convex polygonal barriers and developed solution procedures based on decomposition of the original problems into smaller convex sub-problems. However, the partitioning method becomes computationally expensive as the number of regions and their vertex counts increase. Ref. [7] studied the (generalized) RLocP with congested regions having fixed traveling cost, called the RLocP-CR, and provided metaheuristic solution approaches and extensive computational results on the RLocP test instances in the literature and a set of new test instances.

In this study, we consider the single facility RLocP with congested regions having entry and exit points (*RLocP-CR-EE*) under the Euclidean distance norm. Congested regions are defined as line segments or *arbitrary shaped polygonal* areas on the plane inside which facility location is infeasible. Traveling through the regions is limited to certain direct passages with entry and exit gates. Any traveling through such passages has a certain *fixed* cost. Real-life examples of the defined congestion regions include urban zones where traveling can only occur through some roads, highways, or tunnels. Besides, some factors like congestion on transportation networks, authorization, vehicle modification, and air pollution which increase travel time, risk, or cost, can be coped with the fixed traveling cost on the region.

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The solution approach used in this study is based on evolutionary algorithm (EA) enhanced with variable neighborhood search (VNS). Metaheuristics are commonly used to tackle optimization problems that are difficult or computationally expensive to solve for exact solutions. The RLocP literature includes very few attempts in using metaheuristics. Ref. [4] was the first study that used simulated annealing to solve the RLocP with barriers. However, they provided very limited experimental results and no information about their proposed metaheuristic. A genetic algorithm (GA) is implemented by [14] to find candidate sub-problems introduced by [13] in solving the RLocP with convex polygonal barriers. This approach is different from ours as we solve the original problem directly by the proposed metaheuristic instead of partitioning. Our approach is similar to [7] where the modified EA with VNS is implemented to solve the RLocP with congested regions.

The study is organized as follows. Section 2 provides mathematical formulation of the RLocP-CR-EE and its connections to the known RLocPs in the literature. Solution procedure is explained in Section 3. Section 4 presents computational experiments and their results. We conclude the study in Section 5.

2. The RLocP-CR-EE

2.1. Mathematical formulation

The problem is about locating a single facility on the plane to serve M customers. The set of customer locations is $\mathcal{M} = \{X_1, \ldots, X_M\}$ where each customer m has a nonnegative weight w_m, $m = 1, \ldots, M$. They are also called demand points. There exists a set of disjoint polygonal congested regions as R, where each region $r \in R$ has a bounded interior $B_r, B_r \subset \mathbb{R}^2$. Any region r has an associated nonnegative fixed traveling cost c_r, and a set of vertices U_r, which also includes the entry and exit points. The set of passages in region r is defined by their end points as $P_r = \{(X_i, X_j) | X_i, X_j \in U_r\}$. Traveling on a passage occurs through its extreme points, i.e. either the traveler enters the passage at its entry point and goes all the way to the exit point or it travels on the region border. Facility location inside the regions and on their passages is forbidden. Therefore, the feasible set of location points is defined as $F = \mathbb{R}^2 \setminus \bigcup_{r \in R}\{B_r \cup P_r\}$. The set of all region vertices and all points are given by $\mathcal{V} = \bigcup_{r \in R} U_r$ and $\mathcal{N} = \mathcal{M} \cup \mathcal{V}$, respectively. Members of $\mathcal{V}$ are indexed from $M+1$ to $M+V$ and those of $\mathcal{N}$ are indexed as $1, \ldots, N$, where V corresponds to the total number of region vertices and $N = M+V$ is the total number of points in the problem instance. Finally, the facility location is indicated by a two dimensional vector of its coordinates on the plane as $X_f = (x_f, y_f)$.

In order to formulate the problem, we use the *visibility graph* in [6]. The graph is structured by the set of points $\mathcal{N} \cup \{X_f\}$, defined over an index set $K = \{1, \ldots, N, f\}$, and the set of arcs between any two *visible* points. Two locations are visible to each other if their direct access path does not intersect any $B_r, r \in R$.

Fig. 1 shows a visibility graph on a small problem instance with three demand points $(X_i, i = 1, 2, 3)$ and one congested region $(X_v, v = 4, \ldots, 10)$ with two passages. Lines correspond to a visible connections between associated two pair of points and dashed lines show the passages. The cost of traveling on the links between the gate vertices (X_5, X_{10}) and (X_6, X_9) is increased with additional fixed cost c_r assigned to the region.

Let $E(X_i, X_j)$ be the minimum cost of going from location X_i to location X_j, $i, j \in K$. Then, the RLocP-CR-EE is formulated below, similar to [7]:

$$\min \quad Z(X_f) = \sum_{m=1}^{M} w_m E(X_f, X_m) \tag{1}$$

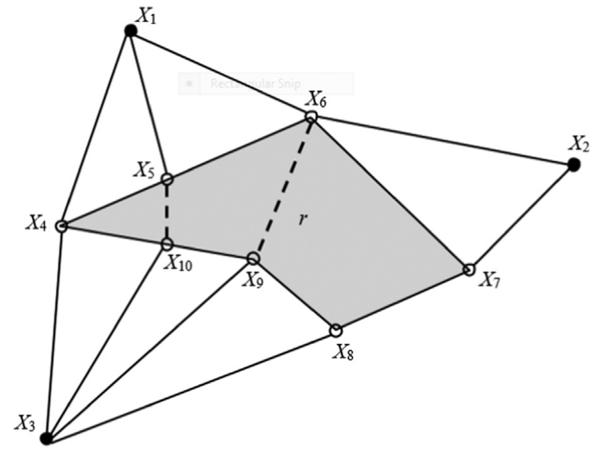

Fig. 1. A visibility graph example with three customers and a congested region with two passages.

$$\text{subject to} \quad X_f \in F \tag{2}$$

The total weighted traveling cost between the facility location and the demand points is minimized by the objective function (1). Eq. (2) ensures that the facility location is feasible. The function $E(X_i, X_j), \forall i, j \in K$, shortly written as E_{ij}^V, is defined recursively by

$$E_{ij}^0 = d_{ij} \tag{3}$$

$$E_{ij}^k = \min\left(E_{ij}^{k-1}, E_{i,M+k}^{k-1} + E_{M+k,j}^{k-1}\right), \quad 1 \le k \le V \tag{4}$$

E_{ij}^k is the *least-cost way* between X_i and X_j, $\forall i, j \in K$, for which all intermediate points are in the set $\{X_{M+1}, \ldots, X_{M+k}\} \subseteq \mathcal{V}$. d_{ij} is called *direct-access cost*, i.e. the cost of the link connecting X_i and X_j in the visibility graph calculated as

$$d_{ij} = \begin{cases} h \times l_2(X_i, X_j) + c_r & \text{if } (X_i, X_j) \in P_r, \forall r \in R \\ h \times l_2(X_i, X_j) & \text{otherwise} \end{cases} \tag{5}$$

where, $l_2(X_i, X_j) = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{1/2}$ is the Euclidean distance function. In cost calculations, it is assumed that one unit distance of traveling costs h monetary units. Without loss of generality, we assume $h = 1$ monetary unit per distance unit.

2.2. Relation of the RLocP-CR-EE with the Existing RLocPs

This section aims to establish connections between the problem formulated in Section 2.1 and the RLocPs in the literature. We refer to [7] for the general RLocP with congested regions, discuss similarities and differences of the RLocP-CR and the RLocP-CR-EE, and address the challenges regarding to the RLocP-CR-EE below.

In the RLocP-CR, traveling is possible through anywhere in the region at an additional fixed cost. If the fixed cost is high enough, traveling may not occur inside the corresponding region. In the RLocP-CR-EE, on the other hand, passing over the regions is forbidden except through certain passages. This can be ensured in the RLocP-CR if the restricted regions in the RLocP-CR-EE is decomposed into several congested regions. To be more clear, consider the congested region in the problem instance illustrated in Fig. 1. Region r can be considered as separate congested regions joint at the passage locations as depicted in Fig. 2. Regions r_1 and r_2 in this figure are separated by the passage (X_7, X_8) in the original region and regions r_2 and r_3 are separated by passage (X_{13}, X_{14}) in the original region shown in Fig. 1. The fixed cost of segmented regions is set to a high positive value (infinity) to ensure no traveling inside them. Thus, such regions become barriers as they are known in the literature. The passages are therefore expressed as linear congested regions with the same fixed traveling cost as