Computers & Operations Research 62 (2015) 224–236

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

A genetic algorithm for the uncapacitated single allocation planar hub location problem

Haluk Damgacioglu*, Derya Dinler, Nur Evin Ozdemirel, Cem Iyigun

Industrial Engineering Department, Middle East Technical University Dumlupinar Bulvari, 06800 Ankara, Turkey

ARTICLE INFO

Available online 18 September 2014

Keywords:
Genetic algorithms
Hub location problems
Planar hub location problems

ABSTRACT

Given a set of n interacting points in a network, the hub location problem determines location of the hubs (transfer points) and assigns spokes (origin and destination points) to hubs so as to minimize the total transportation cost. In this study, we deal with the uncapacitated single allocation planar hub location problem (PHLP). In this problem, all flow between pairs of spokes goes through hubs, capacities of hubs are infinite, they can be located anywhere on the plane and are fully connected, and each spoke must be assigned to only one hub. We propose a mathematical formulation and a genetic algorithm (PHLGA) to solve PHLP in reasonable time. We test PHLGA on simulated and real life data sets. We compare our results with optimal solution and analyze results for special cases of PHLP for which the solution behavior can be predicted. Moreover, PHLGA results for the AP and CAB data set are compared with other heuristics.

1. Introduction

Hub location problems (HLPs) are defined on a network such that there are n interacting points called spokes and p centers of transportation named hubs. The links in a hub location network are of three types. These are: (1) collection links from spokes (origin points) to hubs, (2) inter-hub transfer links between pairs of hubs, and (3) distribution links from hubs to spokes (destination points). The hub location network is first defined in 1969 by Goldman [1]. An example hub location network is shown in Fig. 1 where nodes i and j represent spokes, and k and l are hubs. HLP determines the hub locations and assigns spokes to hubs so as to minimize flow and distance weighted transportation cost in the network.

In a hub location network, the supply from some of the origin points is collected at a hub, transferred to another hub together, and then distributed to the demand points. The main motivation of using a hub-spoke network is to take advantage of the cost reductions between hubs due to the economies of scale. In most cases, the aggregated flow between hubs reduces the total transportation cost compared to direct shipment between all pairs of spokes. Hub-spoke networks are used in various industries such as airlines, shipment, cargo delivery, and telecommunication. For example, since larger trucks are used between hubs, the unit transportation cost decreases in shipping goods. In airline industry and telecommunication networks, use of hubs eliminates the need for all pairwise connections between spokes, reducing the operational costs significantly.

Since hub-spoke networks have many practical uses in various industries, HLP is widely studied by researchers and there are many variants of the problem. For example, each spoke can be assigned to a single hub or multiple hubs, hubs can be uncapacitated or have limited capacity. One variant imposes a constraint on the hub locations such that hubs can only be located on predetermined points, typically some of the origin and destination points. This variant is called the discrete hub location problem (DHLP). If hub locations are not restricted and they can be located anywhere on the plane, the problem is named as the planar hub location problem (PHLP).

Most of the studies in this area focus on solving the discrete version of the problem. DHLP is known to be NP-hard [1]. Therefore, researchers developed some heuristics to find a good solution in reasonable time. DHLP was initially studied by O'Kelly [2]. He proposed the first mathematical programming formulation of the problem and developed two heuristics, namely HEUR-1 and HEUR-2. Klincewicz first developed an exchange heuristic in 1991 [3], and then a Tabu Search and a GRASP heuristic [4]. Campbell [5] introduced two heuristics for the single allocation DHLP. The solutions found by these heuristics are obtained by modifying the solutions for the multiple allocation version of the problem. Moreover, Ernst and Krishnamoorthy [6], and Abdinnour-Helm [7] proposed simulated annealing metaheuristics to solve the problem. In 1998, Ernst and Krishnamoorthy [8] developed a branch

* Corresponding author.
E-mail addresses: hdamgacioglu@gmail.com (H. Damgacioglu), dinler@metu.edu.tr (D. Dinler), nurevin@metu.edu.tr (N. Evin Ozdemirel), iyigun@metu.edu.tr (C. Iyigun).

http://dx.doi.org/10.1016/j.cor.2014.09.003
0305-0548/

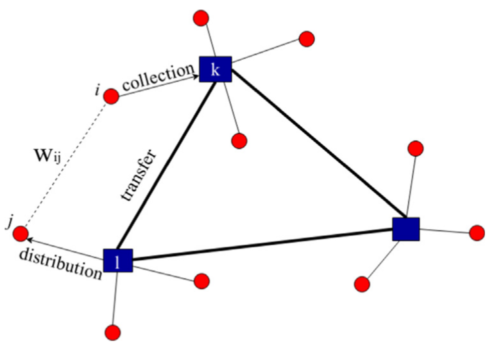

Fig. 1. An example of hub location networks.

and bound algorithm based on the shortest path problem. Topcuoglu et al. [9] and Kratica et al. [10] approached the problem using Genetic Algorithms. Ilic et al. [11] proposed a variable neighborhood search heuristic to solve the problem, which solves instances with up to 1000 nodes in reasonable time.

The planar version of the hub location problem is underappreciated in the literature compared to the discrete version. There are only a few studies on the PHLP. O'Kelly [12] approached the planar hub location based on clustering. He solved instances with up to 500 nodes and 9 hubs. Campbell [13] proposed three strategies to locate the terminals by continuous approximation of freight carrier with increasing demand problem. Aykin and Brown [14] developed a modified location–allocation algorithm to solve the facility location problem in which facilities interact with each other.

Alev and Kara [1], and Campbell and O'Kelly [15] provide detailed reviews of HLP studies.

Our motivation for studying PHLP is threefold. First of all, PHLP is mathematically challenging because it has a non-differentiable objective function, which is not easy to optimize. Secondly, it can be useful in some real world problems where locations can be determined in continuous space, such as city logistics, telecommunications, and cargo delivery systems. For example, a shipping or cargo delivery center within a city can be located at almost anywhere. The same is true for GSM base stations or wireless routers to be located in rural areas. Thirdly, PHLP is more sensitive to problem parameters (amount and cost of flow) than DHLP, and PHLP solutions may provide valuable insight for choosing hub locations in DHLP.

In this study, we give a mathematical formulation of the uncapacitated single allocation PHLP and propose a genetic algorithm to solve it. In this version of the PHLP, all flow between pairs of spokes go through the hubs, capacities of the hubs are infinite, they are fully connected, and each spoke must be assigned to only one hub. The problem is reduced to the multifacility location problem (MFLP) if the costs of inter-hub transfer links are zero, and the costs of collection and distribution links are equal. Megiddo and Supowit [16] have proved that MFLP is NP-Hard. Thus, PHLP is also NP-Hard and a mathematically challenging problem requiring efficient heuristic solution procedures.

To test how our algorithm behaves, we use some simulated data sets representing special cases of PHLP for which the optimal solution can be found. The solution quality of the algorithm is promising and it solves large problem instances in reasonable time. Then, we work on some real world data sets from the literature and compare our results with other heuristics. To the best of our knowledge, we present the first results for the planar versions of the CAB and AP [2,6] data sets. Finally, to show how PHLP solutions can provide insight for DHLP, we compare solutions obtained by modifying the PHLP solutions with the optimal solutions for medium sized DHLP instances. We also present modified PHLP solutions for large DHLP instances.

The rest of the paper is organized as follows. In Section 2, we propose a mathematical programming formulation for the PHLP. Section 3 describes the details of our genetic algorithm. Computational results for simulated and real world data sets are presented in Section 4. Section 5 includes concluding remarks for the study.

2. Mathematical formulation of PHLP

Consider a hub-spoke network consisting of interacting points and three types of links for collection, transfer, and distribution. We assume the following for PHLP.

- The number of hubs is given.
- Hubs are fully connected, and all flow between pairs of spokes go through hubs, i.e. direct shipment between spokes is not allowed.
- Hubs have infinite capacity.
- Each spoke must be assigned to only one hub.

The mathematical programming formulations of PHLP and DHLP are closely related. Therefore, the formulation of DHLP should be primarily investigated to understand and formulate PHLP. We first give the notation commonly used in the formulation of both DHLP and PHLP.

n	The number of nodes (spokes) in the hub-spoke network.
p	The number of hubs.
i,j	Indices for nodes, $i,j=1,\dots,n$.
k,l	Indices for nodes, $k,l=1,\dots,p$.
w_{ij}	Weight representing the amount of flow from node i to node j.
α_c	Collection cost (per unit flow and unit distance) from an origin to a hub.
α_d	Distribution cost (per unit flow and unit distance) from a hub to a destination.
α_t	Transfer cost (per unit flow and unit distance) between a pair of hubs such that $\alpha_t \le \min\{\alpha_c, \alpha_d\}$.
d_{ik}	Distance from node i to node k.

The first mathematical programming formulation of the DHLP was proposed by O'Kelly [2]. In his formulation x_{ik} represents assignment of spokes to hubs for each $i \ne k$, that is x_{ik} takes the value one if node i is assigned to hub k, zero otherwise. When $i=k$, $x_{kk}=1$ means that the node is selected as hub. Then, O'Kelly's [2] mathematical programming formulation of DHLP is as follows.

Problem 1.

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\left\{\sum_{k=1}^{n}\alpha_c d_{ik}x_{ik} + \sum_{k=1}^{n}\alpha_d d_{jl}x_{jl} + \sum_{k=1}^{n}\sum_{l=1}^{n}\alpha_t d_{kl}x_{ik}x_{jl}\right\} \quad (1)$$

$$\text{subject to} \quad (n-p+1)x_{kk} - \sum_{i=1}^{n} x_{ik} \ge 0 \quad \forall k \quad (2)$$

$$\sum_{k=1}^{p} x_{ik} = 1 \quad \forall i \quad (3)$$

$$\sum_{k=1}^{n} x_{kk} = p \quad (4)$$

$$x_{ik} \in \{0,1\} \quad \forall i,k \quad (5)$$

Objective function (1) is a function of the hub selection and assignment decisions. The three terms represent collection costs from spokes to hubs, distribution costs from hubs to spokes, and transfer costs between hubs. Constraint set (2) is used to assign