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Demand point aggregation method for covering problems with gradual coverage

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ABSTRACT

Real world location problems often involve a large number of demand point (DP) data such that the location models become computationally intractable. DP aggregation is a viable means to address the problem by aggregating the original DPs to a smaller set of representative DPs. Most inevitably, though, DP aggregation accompanies a loss of information in the original data and results in errors in the location solution. As such, there is an inherent trade-off between the extent of aggregation and the amount of errors. For covering problems, Current and Schilling (1990) [3] developed an error-free aggregation method based on a key concept that we define in this paper as common reachability set (CRS). While their method provides error-free aggregation solutions to covering problems with binary coverage, it is not applicable to more general and practical cases where the coverage of facilities gradually decreases. We address this limitation by refining the CRS concept. Our method, which we call an approximate CRS (ACRS) method, can be viewed as a generalized version of the original method by Current and Schilling. Using randomly generated DPs data and data from a real world application, we demonstrate the effectiveness of the ACRS method.

1. Introduction

The establishment of service facilities typically requires large investment and has a wide-ranging and lasting socio-economic impact. Thus, selecting locations for new facilities is an important strategic decision. The goal of facility location problems is to determine the optimal locations of servers in order to effectively service demands. The use of mathematical models to solve location problems has a long history dating back to the early 20th century, and location problems have since motivated many operation researchers to develop models and solution algorithms [6,18,38,43].

In many location problems, the demand for service is represented by a finite set of points, referred to as *demand points* (DPs). Real-world location problems often involve a large number of DPs. For example, DPs can be residential addresses in a city, locations of callers requesting ambulances, etc. Since many location problems are NP-hard [30], the number of DPs in a location problem can cause computational intractability. This intractability is an important concern for practical applications of location models.

To maintain computational tractability, one may reduce the size of DPs by *aggregating* the original DPs into a smaller subset. This, of course, comes at a cost: aggregating the original DPs into a subset is inevitably accompanied by a loss of information in the demand data and, consequently, errors in the location solutions. Thus, there is a trade-off between the amount of error due to aggregation and the problem's tractability. If we aggressively aggregate the original DPs to achieve computational tractability, the error due to aggregation may become so large that the quality of the location solution becomes less than desirable. This has led Fotheringham et al. [20] to a strong concern: "…therefore question[s] the reliability of any locational recommendations from a location allocation problem when aggregate demand points are used." An effective DP aggregation scheme should reduce the size of DPs to achieve computational tractability while minimizing the aggregation errors.

In this paper, we present a DP aggregation method that introduces near zero errors particularly for covering problems. We focus on covering problems primarily for two reasons. First, the literature on DP aggregation indicates that relatively little attention has been paid to covering problems, whereas most prior research on DP aggregation deals with p-median problems (see Section 2). Second, Current and Schilling [12] have developed a DP aggregation method for covering problems that does not introduce errors. This method, discussed in Sections 2 and 3 in this paper, uses information on candidate facility sites as well as the locations of DPs to eliminate aggregation errors.

While it provides error-free aggregation solutions, the DP aggregation method proposed by Current and Schilling [12] has a limitation – it requires a binary coverage definition. In many practical applications of

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0305-0548/

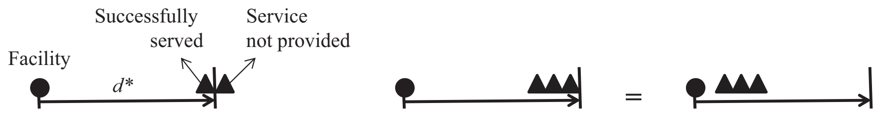

Fig. 1. Two examples where a binary coverage function may not be an appropriate choice for coverage definition. Left: two adjacent demand points are arbitrarily labeled as success vs. failure. Right: advantage (better service quality) provided by a server on the right is not accounted for.

location problem, binary delineation of service coverage can be arbitrary and problematic, and a gradual coverage is often a better description of service coverage. A gradual coverage is used when the coverage quality (or service quality) decreases as a function of distance between the demand point and a facility providing the service. The main advantage of using a gradual coverage is that it allows a location model to directly account for the coverage quality, avoiding arbitrary delineation between a success and failure in service provision. Fig. 1 shows two extreme examples where a binary coverage definition yields an unrealistic and undesirable interpretation.

To address the problem illustrated in Fig. 1, a gradual coverage covering problem was first proposed by Church and Robert [10], and it has been followed by many variants. For example, Berman and Krass [6] proposed a gradual covering problem with a step-coverage function; Karasakal and Karasakal [29] and Eiselt and Marianov [15] developed gradual covering problem in the maximal covering location problem and location set covering problem framework, respectively; Drezner et al. [14] incorporated uncertainty in coverage radius. These models are applied to a variety of location problems where the quality of service perceived at a demand point is a function of the distance from a facility providing the service. Examples include location problems in emergency medical service, telecommunication, and defence applications. In ambulance location problems, it is important that an ambulance arrives at a patient scene as quickly as possible. This time-criticality can be modeled by a survivability function that gradually decreases as the time for an ambulance to arrive at the scene increases. A survivability function has been used in many ambulance location studies, e.g. [9,17,32]. An example in telecommunication applications is a signal transmitters (cell-phone towers) location problem [25,36]. Level of coverage by a cell phone tower is represented by the signal strength at the location, which is a decreasing function of the distance from the tower. Defence applications include missile and radar location problems, where kill probability by a missile attack or detection probability of a radar is a decreasing function of the distance between the target and a missile station or radar location [28,37].

Motivated by the limitation of Current and Schilling's method, we build on a crucial concept introduced by Current and Schilling, which we describe in this paper as a common reachability set (CRS). We generalize the definition of CRS, which allows us to aggregate DPs without any error for a stepwise coverage function case. Then, combining with approximation of a gradual coverage function to a stepwise function, we develop a generalized version of a CRS-based aggregation method. Contributions of this paper are threefold. First, our aggregation method, which we refer to in this paper as approximate CRS (ACRS), can be applied to covering problems with any gradual coverage function. Second, it delivers reliable aggregate DPs as it introduces almost negligible amount of aggregation errors. Third, with the generalization, the central idea of CRS-based aggregation can be applied in other types of location problems, p-median and p-center problems.

This paper is organized as follows. We first present related work on the definition of aggregation errors and the existing aggregation methods in Section 2. Section 3 discusses the details of our proposed method to describe how we generalize Current and Schilling's method. Then, using randomly generated DPs as well as those from a real-world application, we demonstrate the effectiveness of the proposed aggregation method in Section 4, and explain the mechanism behind it in detail in Section 5. Finally we conclude our paper in Section 6

2. Related literature

Research on DP aggregation dates back to the late 1970s. Francis et al. [22] provide a comprehensive review of the subject. Two main questions studied in the literature are: defining a measure for aggregation errors, and developing aggregation methods that reduce aggregation errors.

Several aggregation error measures have been developed, as summarized in Francis et al. [22]. While the distance between DPs and their corresponding aggregate DPs (ADPs) is the most direct measure of aggregation error, many aggregation error measures are defined with respect to a location solution X. For example, the *distance difference error* is defined as the difference between the distance from a DP to the closest server in X and from its corresponding ADP to the server. The reason for comparing DPs and ADPs through a location solution X is that we want to measure aggregation error in terms of its consequences for location solutions. As Francis et al. [22] put it, "the use of ADPs is the *cause* of the aggregation error, but there are error *effects*." The crucial factor in aggregating DPs is whether using ADPs instead of DPs results in a suboptimal location solution. The most direct method for measuring aggregation effects is to consider the objective function value or the location solution. Such measures include the optimality error, the coverage error,[1] and the location error [8,13].

Optimality error measures the quality of an approximate location solution by the extent to which its actual coverage differs from the optimal coverage. It is defined as the difference between the actual objective function value of a solution found using ADPs and the objective function value of the solution found using the original DPs. The latter is the true optimal coverage for the original problem. Often a relative optimality error is reported by dividing the difference by the true optimal objective value.

Coverage error is a measure of how well the approximate location problem – i.e., a location problem with ADPs instead of the original DPs – estimates the original location problem. It measures the difference between the predicted and true coverage values by the location solution obtained from using ADPs. This error is important because over-prediction (under-prediction) of the true coverage can lead to less investment (greater investment) than is actually needed [13].

Location error is a measure of the difference between the true optimal location solution and a location solution by using ADPs, and is the most explicit measure of the effects of DP aggregation. Unfortunately, this concept is not quite practically applicable for a few reasons [22]. For example, measuring the *difference* between two location solutions can be arbitrary when the dimension of the solutions is greater than 2. Also, it is known that the objective function may well be relatively flat in the neighborhood of the true optimal solution [16]. This implies that even when an approximate location solution is *very different* from the true optimal locations, the objective function value from the approximate solution may be close to the true optimal objective value.

In fact, it has been reported that location solutions are much more sensitive than objective function values to DP aggregation [20,24,35]. In other words, location error (however it is defined) tends to be much larger than optimality error. As mentioned in

[1] Coverage error, defined for covering problems by Daskin et al. [13], corresponds to the "cost estimate" error for p-median problems in Casillas [8].