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Strengthening the reliability fixed-charge location model using clique constraints

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A B S T R A C T

The Reliability Fixed-Charge Location Problem is an extension of the Simple Plant Location Problem that considers that some facilities have a probability of failure. In this paper we reformulate the original mathematical programming model of the Reliability Fixed-Charge Location Problem as a set packing problem. We study certain aspects of its polyhedral properties, identifying all the clique facets. We also discuss how to obtain facets of the Reliability Fixed-Charge Location Problem from facets of the Simple Plant Location Problem. Subsequently, we study some conditions for optimal solutions. Finally, we propose an improved compact formulation for the problem and we check its performance by means of an extensive computational study.

1. Introduction

Facility location has become a promising research area within Operations Research and location problems have attracted the interest of many researchers. In general, a facility location problem is concerned with locating a number of facilities and assigning to them a set of customers so that a given objective is optimized. Facility location problems have been applied to several different areas, such us manufacturing, business, telecommunications, health care, construction and defense. For a detailed description of location problems and some applications, readers are referred to [10,23] or [8] and the references therein.

A common assumption in location models is that the open facilities are always available. When this is so, location models endeavor to find solutions which minimize the operating costs i.e., the cost of opening facilities plus the cost of serving customers from these facilities. However, in practice facilities may fail and become unavailable. Consequently, the customers initially assigned to those facilities may be left without service or should be reassigned to those alternative open facilities which have not failed. In any case, the initial operating cost should be increased. The inability to provide service may be due to a wide variety of factors such as inclement weather, labor strikes, change of owner, natural disasters, service maintenance or breakdowns. Reliability location models look for reliable solutions. That is, solutions which cost little more than the optimal solution without taking into account failures but which perform better than when facility failures occur. In general, a reliability location problem involves making three decisions while minimizing the total cost: (i) which facilities (or plants) from a set of potential facilities should be open, (ii) how clients should be assigned to open facilities, (iii) how to deal with clients when open facilities fail, i.e., decide whether to pay the non-service cost or from which other open facility to serve those clients. The uncertainty related to the failure of the facilities can be considered in two different ways: by a stochastic approach which compares different scenarios or by the failure probabilities which are inherent in the facilities. Different works addressing both of these approaches are discussed later in this section. In this paper we follow the second approach.

A wide variety of reliable location problems exists depending on the assumptions made, such as

Number of open facilities: It may or may not be fixed. In the first case, the number of open facilities is exogenous i.e., it is a constant in the problem. Otherwise, the number of open facilities depends on the cost structure, and is an outcome of the problem.

Facility capacity of service: The facilities may be capacitated or not. A capacitated facility has a limited capacity of service, which means that the availability of the product that it serves is limited. On the other hand, a non-capacitated facility has unlimited availability.

Reliability levels: In two-level reliability models if one facility fails, all the customers initially assigned to that facility will be re-allocated to the nearest alternative, which cannot fail simultaneously. Thus, each customer can be assigned to two facilities; one with the lowest cost, which is called the primary facility, and another with the lowest cost when the previous option fails. In multi-level reliability models multiple failures may occur simultaneously and each customer is assigned to an ordered list of facilities. These models usually offer the

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possibility of paying a non-service cost after a certain number of back-up services in order to conclude the process.

Advance information for customers regarding the state of the facility: If such information is provided in advance, customers can travel directly to an operating facility (even though it may not be the facility they would normally use). If, on the other hand, customers do not have information in advance about the state of the facility, they then have to search for the operating facility, possibly by visiting several facilities until they find one able to provide the service. In the first situation it can be said that customers have complete information while in the second case that they have incomplete information.

The set of failable facilities: Sometimes there are facilities that are perfectly reliable and thus never fail, while some others may fail and become unavailable. In this case, we have two different sets of facilities: the set of non-failable facilities and the failable set. We can also consider all the facilities to be subject to possible failures which means that the non-failable set is empty.

Distribution of failure probabilities: The distribution of failure probabilities may or may not be uniform. When all the facilities are similar, it can be assumed that the probability of failure is identical for them all. When facilities are not similar, site-specific probabilities should be considered. Usually, leaving aside the uniformity assumption of facility failure probabilities introduces non-linearity in formulation.

Although the problem of reliability in location has been studied in the literature for years, the work of Drezner [9] was the first to introduce the probability of a facility becoming inactive. The author proposes a heuristic method for solving the problem. Snyder and Daskin [26] were the first to analyze a mathematical model considering uncertainty due to failure probabilities inherent in the facilities. In this paper the authors assume that some facilities are reliable while others are subject to failure, all with the same degree of probability. They formulate the problem as a linear mixed-integer problem and solve it with Lagrangian relaxation.

The assumption of uniform failure probabilities may seem too hard but in the location literature there are many cases of this application. Some examples in which all facilities fail with the same degree of probability are ATM's [3] and traffic surveillance sensors [15,16]. In both cases the common failure probability is obtained from past history performance statistics. Other examples of facilities which can fail with the same degree of probability are hydrants in irrigation channels, cigarette vending machines, coffee machines or telephone booths.

The works of ReVelle et al. [23] and Snyder [25] are two surveys of reliable location problems subsequent to the work of Snyder and Daskin [26]. Later, Berman et al. [2] analyzed several characteristics of optimal location patterns. In particular, the authors study the tendency of facilities to move towards each other as the probability of failure increases. Lee and Chang [13] propose a two-level reliability linear model for non-uniform failure probabilities. However, they perform a computational study with those which are uniform. The PhD dissertation by Zhan in 2007 [28] studies different multi-level reliability linear models for uniform failure probabilities and multi-level reliability non-linear models for site-specific failure probabilities. The author provides several heuristic algorithms and solves large-scale problems. Shen et al. [24] analyze the problem with non-uniform failure probabilities and propose both scenario-based stochastic programming and a non-linear mixed-integer programming model, showing that they are generally equivalent. They also evaluate different heuristic procedures that can produce near-optimal solutions and propose a constant-ratio approximation algorithm for the case where all failure probabilities are the same.

Berman et al. [3] were the first in this field of research to introduce incomplete information to the problem. The authors analyze a multi-level reliability model with uniform failure probabilities and incomplete information. They provide some insights into the behavior of the optimal solutions and develop three heuristic procedures for solving a set of instances. Later, Cui et al. [7] propose a compact multi-level reliability mixed-integer programming formulation for non-uniform failure probabilities and a continuum approximation model to study the reliable uncapacitated fixed-charge location problem: both are solved using a custom-designed Lagrangian relaxation algorithm. Lim et al. [18] deal with non-uniform failure probabilities in a two-level reliability linear model. The authors develop a Lagrangian relaxation solution algorithm. Peng et al. [22] consider a reliable capacitated model where failures can appear in multiple levels. Most recently, Li et al. [14] present two reliable problems, a p-median problem and an uncapacitated fixed-charge location problem. Both models consider heterogenous facility failure probabilities and one layer of supplier back-up and both are formulated as nonlinear integer programming models. The authors develop two Lagrangian relaxation-based solution algorithms.

Due to the complexity of these problems, different authors have also proposed metaheuristic techniques for solving them. Mladenovic et al. [20] present a survey of several metaheuristics for solving p-median problems. Later, Fleszar and Hindi [11] propose a variable neighborhood search metaheuristic. More recently, Alcaraz et al. [1] develop hybrid metaheuristics to solve a reliability location problem.

Recently, the works of Li [15] and Li and Ouyang [16] add new material to the field of facility location with probabilities of facility failure. The authors propose a continuum approximation model for the reliable uncapacitated fixed-charge facility location problem under correlated facility failures and design a reliable traffic surveillance sensor model for uniform failure probabilities. All the advances in the case where all railroad wayside sensors fail with the same probability are incorporated into new software, which has been adopted by the industry [17]. In the context of industry, different authors have also applied reliability location models to the area of supply chain management. Two detailed surveys of these models are presented in [19] and [27].

This paper presents a set packing formulation of the Reliability Fixed-Charge Location Problem (RFLP) originally formulated by Snyder and Daskin [26]. We assume that (i) the number of facilities in the solution is endogenous and depends on the cost structure, (ii) facilities are uncapacitated, (iii) multiple failures may occur simultaneously, (iv) customers have complete information, (v) some facilities may fail while some others may not, (vi) all the facilities fail with the same probability. The paper outlines a number of clique facets that can be added to the formulation as well as some logical constraints that can also be incorporated in the model. A new compact formulation is proposed after testing a number of variants of the model on a set of test problems. In Section 2 we review the original formulation of the Reliability Fixed-Charge Location Problem, proposed by Snyder and Daskin [26]. In Section 3 we study the family of clique facets associated with a set packing formulation of the problem. In Section 4 we present several properties of the optimal solutions. The performance of the improved formulation is tested in Section 5. Finally, some conclusions are outlined in Section 6.

2. Mathematical formulation of the model

Let I be the set of clients and J be the set of potential facility locations. We consider that NF is the subset of facilities in J which are completely reliable and that F is the subset of facilities in J which may fail: $J = F \cup NF$, $F \cap NF = \varnothing$. q is the probability that each facility in F has of failing. For each client $i \in I$ we assume that a demand h_i is required and also that the cost of sending one unit of product from facility $j \in J$ to this client is d_{ij}. The transportation cost is the sum of all these different costs. If all operational facilities are too far away from a