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The firefighter problem: Empirical results on random graphs

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ABSTRACT

The firefighter problem is a deterministic discrete-time model for the spread and containment of fire on a graph. Once the fire breaks out at a set of vertices, the goal addressed in this work is to save as many vertices as possible from burning. Although the problem finds applications in various real-world problems, such as the spread of diseases or hoaxes contention in communication networks, this problem has not been addressed from a practical point of view so far, in the sense of finding a good strategy for the general case. In this work, we develop and compare several integer linear programming techniques and heuristic methods. Random graphs are used for the purpose of comparison. The obtained results shed some light on the challenges for computational tools as caused by graph topology, graph size, and the number of firefighters per iteration, when looking for the best strategy for an a priori unknown graph.
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1. Introduction

The firefighter problem was proposed in 1995 by Hartnell [1] as a deterministic discrete-time model for the spread (and containment) of fire. It has also been used to model several other scenarios such as the spreading of diseases [2,3] and the containment of floods. Existing studies concerning this problem are mostly of theoretical nature, most of them providing results for specific types of graphs. To our knowledge, there is no study so far that addresses this problem for general graphs from a practical point of view, in the sense that the goal is to find a concrete defence strategy to address a particular case without an a priori known topology structure. This is with the exception of two works for specific variants of the problem [3,4], and a preliminary ant colony optimization metaheuristic [5].

The problem is defined as follows. Given an undirected graph $G=(V,E)$, each vertex is initially labeled as *untouched*. At time $t=0$, the fire breaks out at a predefined set of vertices, $B_{init} \subseteq V$. These vertices are labelled as *burnt*. Then, at time step $t=1$, D firefighters (where D is a fixed input parameter of the problem) each choose an untouched vertex from G, and label them as *defended*. Subsequently, the fire is propagated from the burnt vertices to all their neighbors labelled as untouched. Then, these vertices become burnt. This process is repeated iteratively until the fire cannot spread any further, i.e., it is contained.

The optimization objective addressed in this work concerns the selection of the D vertices that are defended at each time step, in order to maximize the number of saved vertices (i.e. nodes that are not burnt) after fire containment. Our aim is to compare the ideas exposed in the literature, in addition to various new ones, in order to study which ones are more appropriate to address concrete cases. We have developed nine integer linear programming (ILP) techniques, derived from the mathematical formulation of the problem provided in [6], and six heuristic approaches, combining the ideas presented in previous works. We have carried out experiments on random graphs and random geometric graphs. Moreover, four different graph sizes, three density levels, and different numbers of firefighters were considered. The results are analyzed globally and separately by topology, size, density and the number of firefighters. Our main results, which indicate ways for obtaining good defence strategies for previously unknown graphs and a limited set of firefighters, are as follows:

- ILP techniques can be adjusted to often provide the best results for the considered benchmarks (ranging from 50 to 1000 nodes, from an average node degree of 7.5–12.5, and from random graphs to random geometric graphs) in a limited computational framework ($|V|/2$ CPU seconds per run). This means that the considered graphs are sufficiently small for ILP techniques to be able to generate and operate on all the necessary variables and constraints (and the associated space of solutions) from the beginning of the process.
- Concerning the considered benchmark, the bottleneck for the ILP techniques is the estimated upper bound of time steps

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needed to contain the fire. Our particular results show that when this number is superior to 20, simpler techniques, such as, heuristic-based greedy methods or truncated ILP approaches may be more suitable for getting good defence strategies.

- Even though saving threatened nodes—that is, those ones that are one step from the fire—is not an optimal heuristic for general graphs, its application is crucial to obtain good defence strategies in the context of the considered random graphs. This suggests that the condition that makes this heuristic suboptimal is not very frequent in random graphs.

The rest of the paper is structured as follows. Section 2 provides an overview on related work. Section 3 presents the most general mathematical formulation of the problem. Section 4 describes the main ideas to tackle the firefighter problem from a practical point of view. These are the foundations for the design of the heuristic approaches. Section 5 presents the experiments carried out. Finally, Section 6 concludes the work.

2. Progress on the firefighter problem

In this section, we provide a short survey on the work developed for the firefighter problem, which, contrary to the present study, is primarily focused on theoretical analysis. Finbow and MacGillivray provide an excellent review in [7].

The associated decision version of the problem, saving just one vertex per time unit, was proved NP-complete for bipartite graphs in 2003 [8]. Stronger results appeared afterwards for cubic graphs [9] and graphs with degree three [10]. More recently, Cygan et al. analyzed the complexity of different parametrized versions of the problem on general graphs [11], and Bazgan et al. [12] and Costa et al. [13] analyzed the case with more than one firefighter.

In 2000, a greedy algorithm for trees, which defends the vertex v that maximizes the number of vertices that will be saved if v is protected, was proved to be a 1/2-approximation algorithm [14]. A linear programming relaxation for trees that supposedly results in a c-approximation algorithm was presented in 2006 [15], and a subexponential $(1-1/e)$–approximation method in 2008 [16]. These results were improved in 2011 [17]. On the other hand, exact polynomial solutions exist for caterpillar and P-trees [7,8,18].

First approaches for grids of dimensions 2 and 3 were provided in 2002 [19,20], and then generalized in 2007 [6]. These studies concluded that two firefighters were needed to contain the fire in an infinite 2-dimensional square grid, and $2d-1$ in a d-dimensional one with $d \geq 3$. Besides, there exist concrete results for triangular, strong, and hexagonal grids [19,21–23], and for other graph classes [24].

The *surviving rate* of a graph is defined as the average percentage of vertices that can be saved when f fires break out at random vertices of the graph [25]. The study of this concept has become very fruitful in the literature and the evidence is the existence of many works on the subject for different graph structures [26–35].

Finally, there are several variants of the firefighter problem, such as the fractional firefighter [19], the spreading vaccinations model [36–38], and the non-constant firefighter problem [39,40]. For more problem variants we refer the interested reader to Section 8 of [7].

3. An integer linear programming model

An ILP model for the classic firefighter problem, aimed at saving as many nodes as possible, was provided in [6]. The formulation, shown in Fig. 1, is based on two sets of binary variables. The first set consists of a binary variable $b_{v,t}$ for each vertex $v \in V$ and each time step $0 \leq t \leq T$, where T is an upper bound for the fire containment process, i.e, the maximum number of iterations needed to contain the fire. A setting of $b_{v,t}=1$ means that vertex v is labelled burnt at time step t, while the opposite indicates that the node is not burnt at that time step. The second set also contains a binary variable $d_{v,t}$ for each vertex $v \in V$ and each time step $0 \leq t \leq T$, and $d_{v,t}=1$ means that vertex v is labelled defended at time step t.

$$
\begin{aligned}
&\max\ |V| - \sum_{v \in V} b_{v,T} \\
&\text{subject to:} \\
&b_{v,t} + d_{v,t} - b_{v',t-1} \geq 0 \quad \forall v \in V,\ \forall v' \in N(v),\ \forall t \in \mathbb{N}, 1 \leq t \leq T \\
&b_{v,t} + d_{v,t} \leq 1 \quad \forall v \in V,\ \forall t \in \mathbb{N}, 1 \leq t \leq T \\
&b_{v,t} - b_{v,t-1} \geq 0 \quad \forall v \in V,\ \forall t \in \mathbb{N}, 1 \leq t \leq T \\
&d_{v,t} - d_{v,t-1} \geq 0 \quad \forall v \in V,\ \forall t \in \mathbb{N}, 1 \leq t \leq T \\
&\sum_{v \in V} (d_{v,t} - d_{v,t-1}) \leq D \quad \forall t \in \mathbb{N}, 1 \leq t \leq T \\
&b_{v,0} = 1 \quad \forall v \in B_{init} \\
&b_{v,0} = 0 \quad \forall v \in V \setminus B_{init} \\
&d_{v,0} = 0 \quad \forall v \in V \\
&b_{v,t}, d_{v,t} \in \{0,1\} \quad \forall v \in V,\ \forall t \in \mathbb{N}, 1 \leq t \leq T
\end{aligned}
$$

Fig. 1. Linear integer programming model of the firefighter problem.

Constraints (2) ensure the spread of the fire while respecting defended vertices. In this context, $N(v)$ refers to the set of vertices connected with v. These constraints imply that, if there exists any $v' \in N(v)$ burnt at iteration $t-1$, then, v must be burnt or defended at iteration t. Constraints (3) prevent a firefighter from defending a burnt vertex and the fire from burning a defended vertex. Constraints (4) ensure that a burnt vertex remains burnt, and Constraints (5) that a defended vertex remains defended. Constraints (6) limit the number of firefighters per time step to D, and Constraints (7)–(9) fix the initial conditions for time step $t=0$, i.e., they label the vertices in B_{init} as burnt and prevent any node from being declared as defended. Without loss of generality, we will assume that the number of vertices initially burnt is only one from now on ($|B_{init}|=1$).[1]

When solving the problem with this model for a specific graph G, a value for parameter T has to be passed to the employed ILP solver. However, providing an adequate value for T is not trivial. Moreover, the value of T has a significant impact on the quality of the solutions obtained within a limited computational environment. If T is underestimated, the solver might not be able to provide a defence plan that contains the fire; and large overestimations lead to the consumption of many unnecessary computational resources. In the following we define the minimal sufficient value of T, denoted by T_{suff}, to be the value such that for $T=T_{\text{suff}}$ the optimal solution is reached with the minimal amount of computational resources. This means that with $T=T_{\text{suff}}$, the fire is contained and the maximal number of nodes is saved. In the following we answer the question of what happens in case $T \neq T_{\text{suff}}$:

- If $T < T_{\text{suff}}$, two possible scenarios may arise. First, it might not be possible to contain the fire. Second, while it is possible to contain the fire, the number of saved nodes might not be maximal (see Fig. 2 for an example with $T=2$ and $T_{\text{suff}}=3$).

[1] Note that all cases in which $|B_{init}|$ is greater than one can easily be transformed to a case in which the fire breaks out in a single vertex. This is done by introducing a *dummy node*, connecting this node to all nodes in B_{init}, removing all nodes from B_{init} and placing the dummy node in B_{init}. Moreover, firefighters must be prevented from defending any node in the first iteration ($d_{v,1}=0 \forall v \in V$).

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