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Mathematical programming algorithms for bin packing problems with item fragmentation

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ABSTRACT

In this paper we consider a class of bin packing problems from the literature having the following distinctive feature: items may be fragmented at a price. Problems of this kind arise in diverse application fields like logistics and telecommunications, and have already been extensively tackled from an approximation point of view. We focus on the case in which splitting produces no overhead, a fixed number of bins is given and the number of fragments or fragmentations needs to be minimized. We first investigate the theoretical properties of the problem. Then we elaborate on them to devise mathematical programming models and algorithms, yielding both exact optimization algorithms and effective heuristics. An extensive experimental campaign proves that our approach is very effective, and highlights which features make an instance computationally harder to solve.

1. Introduction

Due to their generality and practical relevance, packing problems represent one of the most fundamental, and still active, research fields in combinatorial optimization.

In the classical Bin Packing Problem (BPP), that is the most traditional variant, a set of bins of limited capacity and a set of items of known weight are given, and the task is to assign items to bins without splits, in such a way that the sum of weights of items in the same bin does not exceed the bin capacity, and as few bins as possible are used.

Due to their simple structure, BPPs have been one of the first benchmarks for decomposition and column generation algorithms [1], becoming a foundation brick for state-of-the-art methods. Later, books like [2,3] and surveys like [4] contributed to make packing models and algorithms popular in the operations research community. More recent investigations on column generation for BPPs allowed to develop advanced methodologies like dual cuts [5] and generic branching schemes for branch-and-price [6]. State-of-the-art decomposition algorithms can now successfully tackle involved BPPs [7].

Packing problems are of particular relevance also in terms of direct applicability, as in logistics items often correspond to transportation requests placed by customers, and bins correspond to vehicles trailers: a BPP arises whenever a tactical fleet sizing problem has to be faced [8]. Direct application in telecommunication planning is also worth mentioning, where items correspond to data transfer requests, bins correspond to transmission channels, and network dimensioning must be carried out [9].

Of course, stronger market competition and new technologies push for more elaborated methods, asking for more sophisticated models that may be able to capture more application details and provide more optimization power. This is the case, for instance, of Split Delivery Vehicle Routing Problems (SDVRP) in transportation [10] and Fully Optical Network Planning Problems (FONPP) in engineering [11].

The SDVRP models are often applied when capacity is a critical resource: they allow the goods of a single customer to be split and partially loaded into different vehicles, at the price of an increased tracking complexity. Furthermore, while in traditional VRP models each customer needs a single delivery visit, in the SDVRP multiple visits may be needed for some customers, and this is perceived as loss in the level of service. From the tactical point of view, a decision maker is therefore concerned in assessing the fleet size needed to keep the number of splits limited, or in finding plans minimizing the splits.

Similarly, in the FONPP, networks are composed by two types of nodes: edge and core. Each edge node is connected to a single core node through a set of optical fibers, exploiting frequency multiplexing. Connection requests arise between pairs of edge nodes: these are first routed from the source edge to the corresponding core, then to the destination core through the backbone, and finally from the destination core to the destination edge. The more a connection request is split between different fibers, or multiplexed between different frequencies, the more time and energy is

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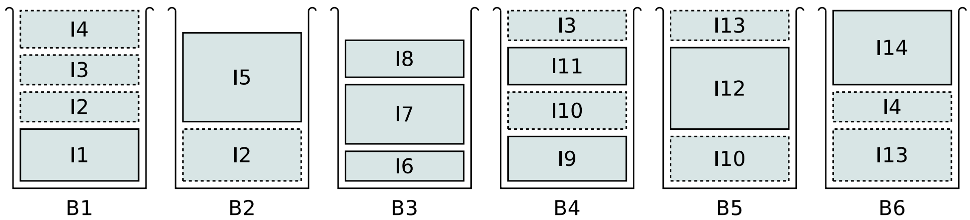

Fig. 1. Instance with $|I| = 14$ and $|K| = 6$. The fragmented items are 2, 3, 4, 10 and 13.

lost in multiplexing and demultiplexing, and in switching at the cores. Hence, finding assignments that minimize the number of splits is crucial, as these require either very complex devices to be used, or signal conversions from the optical to the electrical realm to be performed, thereby substantially losing the advantage of implementing cutting-edge technology.

In the SDVRP customer goods can be seen as items, and vehicles as bins; similarly, in the FONPP connection requests can be seen as items, and pairs optical fiber – multiplexing frequency as bins. In both cases, a packing is needed, taking into account the number of splits. The BPP with Item Fragmentation (BPPIF) has been introduced in the literature to suit these crucial optimization needs, allowing items to be split at a price. Unfortunately, the possibility of splitting substantially complicates BPP models, introducing fractional decisional options that change the nature of the problem, and thereby making not obvious any adaptation of BPP methods from the literature.

Being of key importance also in message transmission in community TV networks, VLSI circuit design [12] and preemptive scheduling on parallel machines with setup times/setup costs, the BPPIF has been first addressed in [13]: the authors proved it to be NP-Hard and discussed the approximation properties of traditional BPP heuristics. In [14] they improved their approximation results, presenting dual asymptotic fully polynomial time approximation schemes that represent the state-of-the-art in approximately solving BPPIFs. Similar models have been introduced in the context of memory allocation problems by [15] and considered in [16]: BPPs are presented in which items can be split, but each bin can contain at most k parts of items; the problem complexity is discussed for different values of k, and simple approximation algorithms are given. Such approximation results have been refined in [17], where the authors provide efficient polynomial-time approximation schemes, and consider also dual approximation schemes.

For what concerns capacity consumption, two variants of the BPPIF are discussed in the literature: the BPPIF with Size Increasing Fragmentation (BPP-SIF), in which the weight of each fragment is increased by a certain amount after splitting, that is splitting introduces overhead, and the BPPIF Size Preserving Fragmentation (BPP-SPF), where the sum of fragment weights equals that of the original item, that is no overhead is introduced by splitting. Instead, for what concerns the objective of the optimization, the BPPIF arises in both bin-minimization form, in which an upper limit on the total number of fragmentations is imposed, and in fragment-minimization form, in which an upper bound on the number of available bins is given. Other variations might be pertinent in practice, including the minimization of bins including fragmented items, and the minimization of fragmentations or fragmented items.

To the best of our knowledge, none of these variants has been tackled from an exact optimization point of view, nor mathematical programming approaches have been explored. In this paper we focus on the fragment-minimization and fragmentations-minimization versions of the BPP-SPF. In fact, these variants are of particular interest, arising in cutting-edge technology optimization [11], and being at the same time harder than their bin-minimization counterparts from a computational point of view [18].

Our aim is threefold. First, we investigate BPP-SPF theoretical properties. Second, we provide exact optimization algorithms by exploiting decomposition techniques; these yield to a column generation framework which is then enriched with problem-specific components like ad-hoc pricing, branching, feasibility check and problem reduction routines. Third, we design effective heuristics. In Section 2 we report our theoretical findings; in Section 3 we discuss both compact and extended mathematical programming formulations for the problem; in Sections 4 and 5 we describe the main ingredients of our algorithms, and a set of performance improvement techniques, respectively. In Section 6 we perform an experimental analysis of our algorithms, and we compare to a state-of-the-art optimization package. Finally, in Section 7 we summarize our results and collect some brief conclusions.

Some of our results have been presented in preliminary forms in [19,18].

2. Properties

We begin by formalizing the problem, and by presenting some structural properties of the BPP-SPF. The key result is that the search for optimal solutions can be pursued, without loss of optimization power, by considering only solutions having very particular structure, thereby reducing the search space.

2.1. Problem definition

It is given a set of items I and a set of bins K. Let w_i be the weight of each item $i \in I$ and let C be the capacity of each bin $k \in K$. The BPP-SPF can be stated as the problem of packing all the items in I into the bins of K. Each item can be split into *fragments* and fractionally assigned to different bins. The aim is to perform the packing in such a way that the sum of the weights of the (fragments of) items packed into a single bin does not exceed the capacity C, minimizing the overall number of fragmentations.

Definition 2.1. A solution to the BPP-SPF is a function $\mu : I \times K \rightarrow [0,1]$ indicating the fraction of each item i packed into each bin k.

Let us define as W_k the amount of space used in bin k, that is

$$W_k = \sum_{i \in I} w_i \cdot \mu(i,k)$$

and as $\overline{W}_k$ the residual space

$$\overline{W}_k = C - W_k$$

A solution μ is feasible if (a) $\overline{W}_k \geq 0$ for each $k \in K$ and (b) $\sum_{k \in K} \mu(i,k) = 1$ for each $i \in I$.