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Interactive minimax optimisation for integrated performance analysis and resource planning

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ABSTRACT

Analysing performances for future improvement and resource planning is a key management function. *Data Envelopment Analysis* (*DEA*) provides an analytical mean for performance modelling without assuming parametric functions. *Multiple Objective Optimisation* (*MOO*) is well-suited for resource planning. This paper reports an investigation in exploring relationships between *DEA* and *MOO* models for equivalent efficiency analysis in a *MOO* process. It is shown that under certain conditions minimax reference point models are identical to input-oriented dual *DEA* models for performance assessment. The former can thus be used for *Hybrid Efficiency and Trade-off Analyses* (*HETA*). In this paper, these conditions are first established and the equivalent models are explored both analytically and graphically to better understand *HETA*. Further investigation in the equivalence models leads to the modification of efficiency measures and the development of a minimax reference point approach for supporting integrated performance analysis and resource planning, with the *Decision Maker*'s (*DM*) preferences taken into account in an interactive fashion. Both numerical and case studies are conducted to demonstrate the proposed approach and its potential applications.

1. Introduction

DEA and *MOO* are tools in management control and planning and to an extent have been developed separately for several decades, with the former directed to analysing past performances as part of management control function and the latter to planning future performances [8]. As a performance measurement and analysis technique, *DEA* is a non-parametric frontier estimation methodology based on linear programming for evaluating the relative efficiency of a set of comparable *Decision Making Units* (*DMUs*) that share common functional goals. *DEA* has evolved tremendously and has been researched extensively since the original work of Charnes et al. [6].

DEA and *MOO* have much in common, though they retain their own distinctive traits [1,5,14,23,24,]. For instance, Doyle and Green [10] suggested that *DEA* is a *Multiple Criteria Decision Analysis* (*MCDA*) method itself. Belton and Vickers [4] and Stewart [24] described similarities between the formulations of basic *DEA* models and classical multi-attribute value function approaches. Sarkis [21] termed *DEA* as a reactive approach to *MCDA* where different alternatives are evaluated objectively. In particular, Golany [11,12] developed an interactive model to allocate a set of input levels as resources and to select the most preferred output levels from a set of alternative points on the efficient frontier. Athanassopoulis [2,3] used goal programming and *DEA* to support resource allocation. Post and Spronk [19] combined the use of *DEA* and interactive goal programming to adjust the upper and lower feasible boundaries of the input and output levels. Joro et al. [14] showed the structural similarity between *DEA* and multiple objective linear programming for value efficiency analysis [13,15–17]. The above techniques require *prior* preference information and/or lead to variation from or restriction on the efficient frontier of an initial *DEA* model, although a hybrid approach was proposed for performance improvement without requiring *prior* preferences or changing an efficient frontier [28,29].

The attraction of *DEA* is that it provides an analytical means for performance modelling without assuming parametric functions between inputs and outputs and an efficient frontier is formulated on which a *DMU* can plan its resources and set its future improvement targets proven achievable by its peers. It is therefore important that the *DM* of the *DMU* can explore the same efficient frontier in order to identify his most preferred performance target in comparison with a fixed benchmark embedded in an initial *DEA* model. Such exploration would be desirable if conducted in an interactive and consistent manner as the *DM* can have an opportunity to investigate and learn what efficient solutions are available and what resources need to be consumed to get to his most

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preferred solution, so that a well-informed decision can be made without having to assume overall preferences *a priori*.

This paper reports an investigation into exploring relationships between *DEA* and *MOO* models for integrated efficiency analysis and resource planning. It is shown that minimax reference point models are identical to input-oriented dual *DEA* models under certain conditions. This equivalence relationship means that the former can be used for *HETA* on the same efficient frontier, so that performance targets can be set and required resources can be planned in an integrated and consistent manner, with the *DM*'s preferences taken into account in an interactive fashion. In this paper, computational studies are conducted to analyse efficiency analytically and graphically in an *MOO* process, leading to the construction of new efficiency measures for *HETA*. Based on the equivalent reference point models, a computational procedure is proposed to analyse data envelope and efficient frontier for interactive trade-off analysis and informed search for the most preferred performance targets. An interactive approach is then explored for *HETA*. Numerical examples and a case study are examined to demonstrate *HETA* and its potential application.

The remainder of the paper is organised as follows. Section 2 briefly presents typical input-oriented *DEA* models as an analytical means for performance modelling, and minimax reference point models as a basis for resource planning. In Section 3, an interactive minimax reference point approach is investigated for *HETA*. Section 4 reports a case study on supplier performance analysis to illustrate the interactive approach. The paper is concluded in Section 5.

2. Analytical models for performance analysis and resource planning

DEA was initially developed by Charnes, Cooper and Rhones in 1978 for measuring and analysing the relative efficiencies of comparable *DMU*s with incommensurate inputs and outputs. In *DEA*, an efficient frontier is formulated, separating efficient *DMU*s from inefficient ones. An efficient *DMU* means that no other *DMU* can either produce the same outputs by consuming fewer inputs, known as the input-orientated approach, or produce more outputs by consuming the same inputs, known as output-orientated approach. In this paper, we first briefly present *DEA* as a non-parametric means for performance modelling.

2.1. Input-orientated DEA models as a means for performance modelling

The original *DEA* model proposed by Charnes et al. [6] is a fractional non-linear programming model, known as the *CCR* model. The objective function in the model is to maximise the single ratio of weighted outputs over weighted inputs for a particular *DMU*, referred to as an observed *DMU* and denoted by DMU_0. Since the fractional model is non-linear and difficult to solve, it is transformed into equivalent linear programming models. In this section, the input-oriented dual *CCR* model is presented as an analytical means for non-parametric performance modelling.

Suppose there are n *DMU*s, each producing s outputs denoted by y_{rj} (the rth output of *DMU j*) and consuming m inputs denoted by x_{ij} (the ith input of *DMU j*). The input-oriented dual *CCR* model is then defined as follows [7]:

$$\begin{aligned} \text{Min} \quad & \theta_{j_0} \\ \text{s.t.} \quad & \theta_{j_0} x_{ij_0} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0, \quad i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rj_0}, \quad r = 1, \ldots, s; \ \lambda_j \geq 0 \text{ for all } j \end{aligned} \tag{1}$$

In the above model, λ_j is a reference parameter for *DMU j* ($j=1, \ldots, n$) and $\lambda_j > 0$ means that *DMU j* is used to construct a composite *DMU* as a benchmark for the observed DMU_0. At the optimal solution of model (1), an optimal composite *DMU* is constructed that cannot underperform DMU_0, which is referred to as the *DEA* efficient solution or the benchmark of DMU_0 in this paper. The benchmark consumes at most a proportion θ_{j_0} of the inputs of DMU_0 and produces at least the same outputs as DMU_0. $\theta_{j_0}=1$ represents a full efficiency score and $0<\theta_{j_0}<1$ reveals the presence of inefficiency. The parameter θ_{j_0} indicates the degree to which DMU_0 has to reduce the consumption of its resources (inputs) in order to become efficient. The reduction is employed concurrently to all inputs and results in a radial movement towards the envelopment surface [7]. Note that such a radial movement strategy is embedded in the *DEA* modelling mechanism *a priori* and does not necessarily take management preferences into consideration, so it is technical rather than preferential. In the following sections, we will explore how efficiency analysis can be conducted equivalently in an *MOO* process so that trade-off analysis can be consistently conducted in the same framework to plan resources, with management preferences incorporated in an interactive fashion.

2.2. Minimax reference point models as a basis for resource planning

In the input-oriented dual *CCR* model, an efficiency score is generated for an observed DMU_0 by minimising inputs with outputs constrained as least at their required levels. This is in essence a *MOO* problem. In this section, we briefly describe basic *MOO* concepts and models, in particular the minimax reference point models as a basis for resource planning.

Suppose a *MOO* problem has m objectives to be minimised, in general represented by

$$\begin{aligned} \text{Min} \quad & f(\lambda) = [f_1(\lambda) \cdots f_i(\lambda) \cdots f_m(\lambda)] \\ \text{s.t.} \quad & \lambda \in \Omega = \{\lambda | g_j(\lambda) \leq 0, \ h_l(\lambda) = 0; \ j = 1, \ldots, k_1, \ l = 1, \ldots, k_2\} \end{aligned} \tag{2}$$

where Ω is a feasible decision space, $f_i(\lambda)$ ($i=1,\ldots, m$) are continuously differentiable objective functions, and $g_j(\lambda)$ ($j=1,\ldots, k_1$) and $h_l(\lambda)$ ($l=1,\ldots, k_2$) are continuously differentiable inequality and equality constraint functions respectively. In this paper, $f_i(\lambda)$, $g_j(\lambda)$ and $h_l(\lambda)$ are all assumed to be the linear functions of λ.

In a *MOO* problem, we are interested in finding efficient solutions. A feasible solution λ^* is said to be efficient if there exists no other feasible solution which is better than λ^* at least on one objective and as good as λ^* on all other objectives. An efficient solution can be formally defined as follows.

Definition 1. In formulation (2), a feasible solution $\lambda^* \in \Omega$ is an efficient solution if and only if there does not exist any other feasible solution $\lambda \in \Omega$ such that $f_i(\lambda) \leq f_i(\lambda^*)$ for all $i=1,\ldots, m$ and $f_\tau(\lambda) < f_\tau(\lambda^*)$ for at least one $\tau \in \{1, \ldots, m\}$.

Any efficient solutions of a *MOO* problem can be generated using a minimax formulation [22,27]. Suppose λ is an efficient solution of model (2) and f_i^* is the minimum feasible value of objective i. There exists a weighting vector w satisfying $w_1 = 1$ and $w_i > 0$ for $i=2, \ldots, m$ and a reference point f^{ref} such that λ can be generated by solving the following weighted minimax reference point problem [27]:

$$\begin{aligned} \underset{\lambda}{\text{Min}} \ \underset{1 \leq i \leq m}{\text{Max}} \quad & \{w_i |(f_i(\lambda) - f_i^{ref})|\} \\ \text{s.t.} \quad & \lambda \in \Omega \end{aligned} \tag{3}$$

The weighted minimax reference point formulation will be called the ideal point model if the ideal point $f^* = [f_1^* \ldots f_m^*]^T$ is used as the reference point $f^{ref} = [f_1^{ref} \ldots f_m^{ref}]^T$. In the minimax reference point formulation, for a given weight vector, the *DM* is assumed to