Computers & Operations Research 45 (2014) 87–96

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

A heuristic and a branch-and-bound algorithm for the Assembly Line Worker Assignment and Balancing Problem

Leonardo Borba, Marcus Ritt *

Instituto de Informática, Universidade Federal do Rio Grande do Sul, Av. Bento Gonçalves 9500, Porto Alegre-RS, Brazil

ARTICLE INFO

Available online 17 December 2013

Keywords:
Assembly Line Balancing
Heterogeneous workers
Branch-and-bound
Beam search
MIP

ABSTRACT

In traditional assembly lines, it is reasonable to assume that task execution times are the same for each worker. However, in Sheltered Work Centres for Disabled this assumption is not valid: some workers may execute some tasks considerably slower or even be incapable of executing them. Worker heterogeneity leads to a problem called the Assembly Line Worker Assignment and Balancing Problem (ALWABP). For a fixed number of workers the problem is to maximize the production rate of an assembly line by assigning workers to stations and tasks to workers, while satisfying precedence constraints between the tasks.

This paper introduces new heuristic and exact methods to solve this problem. We present a new MIP model, propose a novel heuristic algorithm based on beam search, as well as a task-oriented branch-and-bound procedure which uses new reduction rules and lower bounds for solving the problem. Extensive computational tests on a large set of instances show that these methods are effective and improve over existing ones.

1. Introduction

The Universal Declaration of Human Rights states that "everyone has the right to work, to free choice of employment, to just and favourable conditions of work and to protection against unemployment" [1]. Despite this, low employment rates still demonstrate the lack of job opportunities for more than 785 million persons with disabilities, including 110 million with a severe deficiency degree, due to factors like prejudices and absence of appropriate technical preparation [2]. This deficit leads to the creation of programs for the social inclusion of persons with disabilities. Some of them concern their qualification [3], while others ensure opportunities by quota laws [4]. Countries like Spain, Japan and Switzerland merged these two forms by creating Sheltered Work Centres for Disabled (SWDs) [5], which employ mainly persons with disabilities and provide training and a first job opportunity for them [6]. SWDs are not-for-profit industries applying all revenues in improvements for the company and the creation of new jobs.

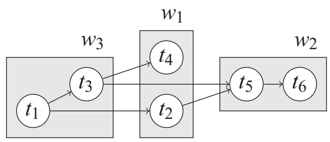
Miralles et al. [6] have shown that using assembly lines in SWDs has advantages, because the division of work into smaller tasks can effectively hide the differences among the workers. Furthermore, the execution of repetitive tasks, when properly assigned, can be an excellent therapeutic treatment for workers with disabilities. Traditional approaches to the optimization of assembly lines assume that the workers have similar abilities and are capable of executing the tasks in the same time. The most basic model of this kind is the Simple Assembly Line Balancing Problem (SALBP), which has been extensively studied in the literature [7]. Several authors have considered stochastic models of assembly lines, where task times may vary, and remedial actions are taken if the cycle time is exceeded at some station [8–13]. In this paper we are not directly concerned with varying task times of a single worker, but with the case of SWDs, where the workers need different times to execute the tasks, or may even be incapable of executing some of them. To model such a situation, Miralles et al. [14] proposed the Assembly Line Worker Assignment and Balancing Problem (ALWABP), which assigns tasks to different workers and these workers to the workstations. This problem is a single line, single-model, basic straight assembly line, with task time attributes depending on the worker attribute of the workstation, occurrence and precedence constraints, minimizing the cycle time, in the taxonomy of Battaïa and Dolgui [15], and classified as $[pa, link, cum|equip|\ c]$ by Boysen et al. [16].

1.1. Problem definition

Let S be a set of stations, W be a set of workers, $|W|=|S|$, and T be a set of tasks. Each workstation $s \in S$ is placed along a conveyor belt and is assigned to exactly one worker $w \in W$, which is responsible for executing a subset of tasks $x_w \subseteq T$. The tasks are

* Corresponding author.
E-mail addresses: lmborba@inf.ufrgs.br (L. Borba), marcus.ritt@inf.ufrgs.br (M. Ritt).

0305-0548/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cor.2013.12.002



p_{tw}	t_1	t_2	t_3	t_4	t_5	t_6
w_1	4	4	3	1	1	6
w_2	∞	5	6	5	2	4
w_3	3	4	2	∞	3	∞

Fig. 1. Example of an ALWABP instance and an assignment of tasks to workers (in gray). Upper part: precedence constraints among the tasks. Lower part: task execution times.

partially ordered, and we assume that the partial order is given by a transitively reduced directed acyclic graph $G(T,E)$ on the tasks, such that for an arc $(t,t')\in E$ task t precedes task t'. Therefore, the station that executes task t cannot be placed later than that of task t' on the conveyor belt. The execution time of task t for worker w is p_{tw}. If a worker w cannot execute a task t, p_{tw} is set to ∞.

The total execution time of worker w is $D_w=\sum_{t\in x_w}p_{wt}$. The cycle time C of the line is defined by the maximum total execution time $\max_{w\in W}D_w$. In assembly line balancing, a problem of *type* 1 aims to reduce the number of stations for a given cycle time. Since in SWDs the goal is to include all workers, our problem is of *type* 2, and aims to minimize the cycle time for a given number of stations and the same number of workers. A valid solution is an assignment of workers to stations together with an assignment of tasks to workers that satisfies the precedence constraints.

Fig. 1 shows an example of an ALWABP-2 instance. For the assignment given in the figure, we have $D_{w_1}=5$, $D_{w_2}=6$, $D_{w_3}=5$, and a cycle time of $C=\max\{D_{w_1},D_{w_2},D_{w_3}\}=6$.

1.2. Related work

The majority of the publications on the ALWABP-2 is dedicated to the application of meta-heuristics to find approximate solutions to the problem. Two clustering search methods were proposed by Chaves et al. [17], Chaves et al. [18], which were outperformed on large instances by a tabu search of Moreira and Costa [19]. Blum and Miralles [20] proposed an iterated beam search based on the station-oriented branch-and-bound procedure of Miralles et al. [14]. Later, Moreira et al. [21] used a constructive heuristic with various combinations of priority rules to produce initial solutions for a genetic algorithm (GA). Mutlu et al. [22] developed an iterated GA that produces valid orders of tasks and applies iterated local search to attribute the tasks in the selected order to the workers.

The only exact published method for the ALWABP-2 is the branch-and-bound procedure of Miralles et al. [14]. It embeds a station-oriented, depth-first branch-and-bound search in a linear lower bound search for the optimal cycle time, and is limited to very small instances. A recent working paper of Vila and Pereira [23] proposes a station-oriented branch-and-bound-and-remember algorithm with a cyclic best-first search strategy for the ALWABP-F, following the approach of Sewell and Jacobson [24] for solving the SALBP-1.

1.3. Structure of the paper

In Section 2 we introduce a new MIP model for the ALWABP-2. In Section 3 we present several lower bounds for the problem. A new heuristic for ALWABP-2 is proposed in Section 4. In Section 5 we present a task-oriented branch-and-bound method for solving the problem exactly. Computational results are presented and analyzed in Section 6. We conclude in Section 7.

2. A mathematical formulation with two-index variables

In this section we will present a new mixed-integer model for the ALWABP-2. Currently, the only model used in the literature, called M_1 here, is the one proposed by Miralles et al. [14]. It has $O(|T||W||S|)$ variables and $O(|T|+|E|+|W||S|)$ constraints. In the following we will use the notation defined in Table 1.

2.1. Formulation with two-index variables

Our formulation is based on the observation that it is sufficient to assign tasks to workers and to guarantee that the directed graph over the workers, induced by the precedences between the tasks, is acyclic. Therefore our model uses variables x_{wt} such that $x_{wt}=1$ if task $t\in T$ has been assigned to worker $w\in W$, and d_{vw} such that $d_{vw}=1$ if worker $v\in W$ must precede worker $w\in W$. In this way, we obtain a model M_2 as follows:

$$\text{minimize}\quad C, \tag{1}$$

$$\text{subject to}\quad \sum_{t\in A_w} p_{tw}x_{wt}\le C,\quad \forall w\in W, \tag{2}$$

$$\sum_{w\in A_t} x_{wt}=1,\quad \forall t\in T, \tag{3}$$

$$d_{vw}\ge x_{vt}+x_{wt'}-1,\quad \forall (t,t')\in E,\ v\in A_t,\ w\in A_{t'}\backslash\{v\}, \tag{4}$$

$$d_{uw}\ge d_{uv}+d_{vw}-1,\quad \forall \{u,v,w\}\subseteq W,\ |\{u,v,w\}|=3, \tag{5}$$

$$d_{vw}+d_{wv}\le 1,\quad \forall v\in W,\ w\in W\backslash\{v\}, \tag{6}$$

$$x_{wt}\in\{0,1\},\quad \forall w\in W,\ t\in A_w, \tag{7}$$

$$d_{vw}\in\{0,1\},\quad \forall v\in W,\ w\in W\backslash\{v\}, \tag{8}$$

$$C\in\mathbb{R}. \tag{9}$$

Constraint (2) defines the cycle time C of the problem. Constraint (3) ensures that every task is executed by exactly one worker. The dependencies between workers are defined by constraint (4): when a task t is assigned to worker v and precedes another task t' assigned to a different worker w, worker v must precede worker w. Constraints (5) and (6) enforce transitivity and anti-symmetry of the worker dependencies. As a consequence of these constraints, the workers of a valid solution can always be ordered linearly.

2.2. Continuity constraints

We can strengthen the above model by the following observation: if two tasks i and k are assigned to the same worker w, then all tasks j that are simultaneously successors of i and predecessors of k should also be assigned to w. These *continuity constraints* generalize constraints proposed by Peeters and Degraeve [25] for

Table 1
Notation for ALWABP-2.

S	Set of stations
W	Set of workers
T	Set of tasks
$G(T,E)$	Transitively reduced precedence graph of tasks
$G^*(T,E^*)$	Transitive closure of graph $G(T,E)$
p_{tw}	Execution time of task t by worker w
$A_w\subseteq T$	Set of tasks feasible for worker w
$A_t\subseteq W$	Set of workers able to execute task t
P_t and F_t	Set of direct predecessors and successors of task t in G
P_t^* and F_t^*	Set of all predecessors and successors of task t in G^*
$C\in\mathbb{R}$	Cycle time of a solution