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Heuristics for the traveling repairman problem with profits

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ABSTRACT

In the traveling repairman problem with profits, a repairman (also known as the server) visits a subset of nodes in order to collect time-dependent profits. The objective consists of maximizing the total collected revenue. We restrict our study to the case of a single server with nodes located in the Euclidean plane. We investigate the properties of this problem, and we derive a mathematical model assuming that the number of nodes to be visited is known in advance. We describe a tabu search algorithm with multiple neighborhoods, we test its performance by running it on instances from the literature and compare the outcomes with an upper bound. We conclude that the tabu search algorithm finds good-quality solutions fast, even for large instances.

1. Introduction

Imagine a single server, traveling at a constant speed. There are n locations given, each with a profit p_i, $1 \le i \le n$. At $t=0$, the server starts traveling and collects a revenue $p_i - t_i$ at each visited location, where t_i denotes the server's arrival time at location i. Not all locations need to be visited. The problem is to find a travel plan for the server that maximizes the total revenue. Notice that when the profits are very large, i.e., $p_i > t_i$, $\forall i$, the problem comes down to finding a tour minimizing the sum of the waiting times; this problem is known as the minimum latency or traveling repairman problem. The problem studied in this paper is the traveling repairman problem with profits (TRPPs). In particular, we perform a computational study of the TRPP in the Euclidean plane.

1.1. Motivation

The TRPP occurs as a routing problem in relief efforts. For example, consider the following situation. In the aftermath of a disaster like an earthquake, there are a number of villages that experience an urgent need for medicine. The sooner the medicine gets to a village, the more people can be rescued. Since the cost of transport is negligible compared to the value of a human life, rescue teams are only concerned with the total number of people that can be saved. Assume that at location i, p_i people are in need of the medicine, and that every instance of time, there is one of them dying. Suppose also that we have one truck available. With t_i denoting the arrival time of the truck at location i, the number of people that will survive equals $p_i - t_i$. Thus, the goal of the rescue team is to maximize $\sum_i (p_i - t_i)$, where the sum runs over all the visited locations. Note that from the moment that t_i becomes larger than p_i for an unvisited location i, that location will not be visited anymore. This situation is described in [6] and is equivalent to the TRPP.

Another, more theoretical, motivation concerns the k-traveling repairman problem (k-TRP). The k-TRP is a traveling repairman problem with multiple servers. All clients need to be visited such that the latency, i.e., the sum of the waiting times, is minimized. Observe that no profits are considered in this problem. As explained in Coene and Spieksma [7], one potential way of solving such a problem is a set-partitioning approach where an integer programming model is built, using a binary variable x_{rk} for each set of clients; this variable is equal to 1 if route $r \in \{1, \dots, R\}$ is served by server $k \in \{1, \dots, K\}$, with R and K the number of feasible routes and the number of servers, respectively. Let c_{rk} be the latency of a feasible route r served by server k. With this notation, the set-partitioning approach looks as follows:

$$\text{Minimize} \quad \sum_{r=1}^{R} \sum_{k=1}^{K} c_{rk} x_{rk}$$

$$\text{subject to} \quad \sum_{r: i \in r} \sum_{k=1}^{K} x_{rk} = 1 \quad \text{for } i = 1, \dots, n,$$

$$\sum_{r=1}^{R} x_{rk} \le 1 \quad \text{for } k = 1, \dots, K,$$

$$x_{rk} \in \{0,1\} \quad \text{for } r = 1 \dots, R \quad \text{and} \quad k = 1, \dots, K.$$

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When applying a branch-and-price approach to the resulting integer program, it can be observed that the so-called pricing problem in this approach is exactly the TRPP, where the dual variables play the role of profits.

Other applications of routing problems with time-dependent revenues are described in Coene and Spieksma [7], Erkut and Zhang [9], Lucena [13], and Melvin et al. [14], which deals with a problem occurring in "multi-robot routing".

1.2. Literature

Several problems are closely related to the traveling repairman problem with profits (TRPP). The TRPP has similarities with the traveling salesman problem (TSP) [3]. However, contrary to the TSP, in the TRPP not all the nodes need to be visited. Further, an optimal TRPP-solution is a path whose course is influenced by the depot location and may contain intersections. Notice that the latter is always sub-optimal for the Euclidean TSP.

Routing problems with profits are, e.g., the TSP with profits (TSPP) [10] and the orienteering problem (OP) [21]. In the latter, a subset of nodes should be selected in order to maximize the profit under a time-constraint. As in the TRPP, an optimal solution may leave some nodes unvisited. Both the total profit and the distance traveled are inserted in the objective function of the TSPP. The TRPP, however, differs in the sense that revenues are time-dependent.

Problems with time-dependency are relevant in many cases. See [13] for the time-dependent traveling salesman problem (TDTSP). In the TDTSP, the travel time between two vertices depends on the arrival time of the server. This is different from the TRPP where the travel time between vertices is constant, but the profit depends on the arrival time. Tang et al. [20] introduced the multiple tour maximum collection problem with time-dependent rewards; a problem in which jobs are to be scheduled over multiple days and in which the reward corresponding to a job is time-dependent. For this problem, the goal is to find multiple tours, and rewards depend on which tour (i.e., which day) the job is assigned to; this is different from the TRPP, where rewards depend on the position of the job in the tour and not all jobs need to be scheduled.

The TRPP is a generalization of the traveling repairman problem (TRP) [5], i.e., an instance of the TRPP with extremely high profits is also an instance for the TRP with the same optimal solution. In the TRP (also known as the minimum latency problem or the delivery man problem), a single server needs to visit all nodes such that the total latency is minimized. In a classical paper by Afrati et al. [2], it is shown that the TRP on the line can be solved in polynomial time by dynamic programming. This result was generalized to the TRPP on the line by Coene and Spieksma [7]. Since the TRP is NP-hard for more general metric spaces, see the argumentation given in [5], and since the TRPP is a generalization of the TRP, we conclude that the TRPP for these general metric spaces, among which the Euclidean plane, is NP-hard.

As far as we know, no computational studies have been performed for the TRPP. In [7], the TRPP on the line is being solved in polynomial time by a dynamic programming algorithm. No other results are known.

Exact algorithms and approximation algorithms for the TRP have been described in Ausiello et al. [4], Goemans and Kleinberg [12], and Wu et al. [22]; a metaheuristic for the TRP is described in the contribution of Salehipour et al. [18]. An extension of the TRP, the cumulative VRP, is dealt with by Ngueveu et al. [16]. Their memetic algorithm can also be used to solve TRP-instances; this is done for comparison in [18]. Note that the algorithm presented in [16] uses a tour-splitting procedure which has no effect in the single-vehicle case of the TRP. For a review of the metaheuristics for other related problems we refer to Feillet et al. [10], Vansteenwegen et al. [21], and the references contained therein. A general description of some metaheuristics, including the ones that are used in this paper, is given in [11] and [19].

This paper is structured as follows. In the next section, the TRPP is described in detail and a mathematical model is given. A tabu search algorithm is presented in Section 3. The data sets are introduced in Section 4 and the computational results are discussed in Section 5. In Section 6, we compare our results for the TRP with the results from the literature. The conclusions of this paper are summarized in Section 7.

2. Mathematical model

Consider a complete undirected graph $G=(V,E)$, where $V=\{0,1,\ldots,n\}$ is the node set, and E is the set of edges. Each node $i\neq 0$ has an associated profit p_i. There is a single server located at node 0, the depot. The time it takes the server to travel from node i to node j is defined by $d_{i,j}$, with, $d_{i,j}$ satisfying the triangle inequality for all i,j. Further, we assume that the time to serve a node is negligible. If the server arrives at node i at time t_i, a revenue of $p_i - t_i$ is collected. As a consequence, an optimal tour will not contain a node i with $p_i \le t_i$. The goal of the TRPP is to select an ordered subset of nodes such that visiting them one by one maximizes the sum of all the revenues. Each node can only be visited once, and the server does not need to return to the depot.

We now derive a mathematical model for this problem when the number of nodes to be visited, k, is given. Thus, k is the number of nodes whose revenue is collected. Notice that k is just an integer that contains no information about which nodes are visited. The optimal solution for the original problem, with k a variable, can then be found by solving the model for each value of k and selecting the best one; we will come back to this issue at the end of this section. Let $K=\{1,2,\ldots,k\}$.

For each $i\in V, j\in V_0 = V\backslash\{0\}$, and $\ell\in K$, we define the variable $y_{i,j,l}$ as follows:

$$y_{i,j,\ell}=\begin{cases}1 & \text{if edge } (i,j) \text{ is used as the } \ell\text{th edge,}\\ 0 & \text{else.}\end{cases}$$

This definition says that if $y_{i,j,\ell}=1$ then (i,j) is the ℓth edge of the path. Hence i is the $(\ell-1)$th and j is the ℓth node that is visited. The depot is node 0. Observe that if $y_{i,j,\ell}=1$, $d_{i,j}$ is counted $k+1-\ell$ times in the total latency. Hence

$$\sum_{i:i \text{ is visited}} t_i = \sum_{\{(i,j,\ell)|y_{i,j,\ell}=1\}} (k+1-\ell)d_{i,j}. \tag{1}$$

Now the mathematical model can be constructed.

With k, the number of nodes visited, given, the mathematical model is the following:

$$\text{Maximize} \quad \sum_{i\in V}\sum_{j\in V_0}\sum_{\ell\in K}(p_j-(k+1-\ell)d_{i,j})y_{i,j,\ell} \tag{2}$$

$$\text{subject to} \quad \sum_{i\in V}\sum_{\ell\in K} y_{i,j,\ell}\le 1 \quad \forall j\in V_0, \tag{3}$$

$$\sum_{i\in V}\sum_{j\in V_0} y_{i,j,\ell}=1 \quad \forall \ell\in K, \tag{4}$$

$$\sum_{i\in V} y_{i,j,\ell}-\sum_{i\in V_0} y_{j,i,\ell+1}=0 \quad \forall j\in V_0,\ \forall \ell\in K\backslash\{k\}, \tag{5}$$