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The synchronized arc and node routing problem: Application to road marking

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ABSTRACT

This paper introduces the synchronized arc and node routing problem, inspired from a real application arising in road marking operations. In this setting, several capacitated vehicles are used to paint lines on the roads and a tank vehicle is used to replenish the painting vehicles. The aim of the problem is to determine the routes and schedules for the painting and replenishment vehicles so that the pavement marking is completed within the least possible time. This must be done in such a way that the routes of the painting and replenishment vehicles are synchronized. An adaptive large neighborhood heuristic is described and evaluated over a large set of artificial instances.

1. Introduction

This work introduces the synchronized arc and node routing problem (SANRP), inspired from a real application arising in road marking operations. A number of applications of the SANRP are encountered in other contexts such as door-to-door delivery of mail, newspapers or promotional material. A common feature of these applications is the existence of multiple capacitated arc routes which require replenishments. In general, replenishments are carried out by a dedicated vehicle. In this work, to make our presentation more concrete, we will describe the SANRP in terms of the road marking application. In this problem, several capacitated vehicles are used to paint lines on the roads and a tank vehicle can replenish the painting vehicles once or several times a day, depending on the quantity of paint they need. This means that the painting vehicles need not return to the depot when they are replenished. As is standard in capacitated arc routing problems [6], some road segments must be painted (they are required), whereas others do not need to be painted (they are non-required). In the SANRP, there are multiple capacitated painting vehicles and a replenishment vehicle initially located at a common depot. The aim is to determine a set of routes for the painting vehicles as well as a route for the replenishment vehicle so that the pavement marking is completed within the least possible time and the painting vehicles never run out of paint. Hence, it is desirable to synchronize the routes of the two vehicle types.

In the SANRP two routing problems must be solved simultaneously: a multi-vehicle capacitated arc routing problem and a node routing problem. The nodes at which the arc routes and the node route intersect are not given a priori, but must be determined together with the routes themselves. Finally, the routes generated should be synchronized so as to reduce the waiting time at the refill nodes. We analyze three replenishment policies: (i) there is no replenishment vehicle and the painting vehicles return to the depot when they need a refill; (ii) the painting vehicles do not return to the depot when they need a refill, but are serviced by the replenishment vehicle; and (iii) a combination of the first two policies, meaning that the painting vehicles can be refilled from the replenishment vehicle or directly from the depot. Policies (ii) and (iii) are compared with policy (i) which is the standard practice.

The SANRP is considerably more difficult to solve than either the CARP or the node routing problem which it integrates. This is so because any change in the solution of one of the two subproblems affects the solution of the other one. It is of course impractical to solve the SANRP exactly for any realistic size. We have designed a powerful adaptive large neighborhood search heuristic (ALNS) in which the solution space is explored by means of several operators. The heuristic was successfully tested over a large set of artificial.

The remainder of the paper is structured as follows. Section 2 summarizes a number of contributions related to the SANRP. Section 3 formally describes the SANRP. The solution procedure is described in Section 4, and computational results are presented in Section 5, followed by conclusions in Section 6.

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2. Related work

Very few studies have been performed on arc routing or vehicle routing problems with synchronization constraints. Amaya et al. [1] have introduced the CARP with refill points (CARP-RP), also in the context of road marking. Their paper considers a problem with a painting vehicle and a replenishment vehicle, which can supply the painting vehicle at any road junction and must return to the depot after each refill. The authors have proposed an integer linear programming formulation and a cutting plane algorithm for this problem and have applied it to instances involving between 20 and 70 nodes, and between 50 and 595 arcs. For larger instances computational times become rather important. An extension of this work, by the same authors [2], is called the capacitated arc routing problem with refill points and multiple loads (CARP-RP-ML). Contrary to what happens in the CARP-RP, in the CARP-RP-ML the replenishment vehicle does not return to the depot after reloading the painting vehicle. The authors describe a heuristic procedure based on the cluster-first-route-second principle. The largest instances solved contain 70 nodes and 600 arcs. Note that the two papers of Amaya et al. [1,2] are rooted in the same context as the SANRP proposed in the present paper, but our problem is more general since it involves several painting vehicles instead of only one, and it considers three replenishment policies. Moreover, in the SANRP the refill nodes location and routing decisions of multiple vehicles are intertwined.

Rosa et al. [16] have introduced a related problem called the arc routing and scheduling problem with transshipment (ARPT). They consider two types of vehicles: small vehicles are used to collect garbage from the streets and to bring it to a transfer station, and large vehicles are used to transport the garbage from the transfer station to a dump site. The ARPT consists of designing a set of routes for the small vehicles and a set of schedules for large ones in order to minimize the total travel time. In this problem, the location of the meeting point (transfer station) for two vehicle types is an exogenous decision, whereas in the SANRP it is endogenous. In addition, in the ARPT the route followed by the large vehicles is predefined; in contrast, in the SANRP the route of the replenishment vehicle must be designed by taking into account the refill nodes chosen for the painting vehicles.

Pia and Filippi [11] have studied a real-life waste collection problem arising in Due Carrera, a town located in Northern Italy. They called this problem the CARP-MD (capacitated arc routing problem with mobile depots). This problem consists of determining meeting points along the vehicle routes so that the smaller vehicles can dump their content into a larger vehicle before resuming their route. In the case study carried out in Due Carrera, a fleet of four vehicles of two types was available to collect the waste: two large ones called compactors, and two small ones called satellites, and there were restrictions on the streets each vehicle type could use. The CARP-MD is related to the SANRP in the following way: each satellite vehicle can be viewed as a painting vehicle and the compactors correspond to the replenishment vehicle. However, there are two main differences between the two problems. First, in the SANRP the replenishment vehicle is exclusively used to refill the tanks of the painting vehicles, which means that it cannot service the streets in the network. In contrast, a compactor in the CARP-MD is used to service the streets and to pick up trash from the satellites. Second, the SANRP is a pure combination of arc routing and node routing problems with synchronization constraints, whereas the CARP-MD has been described as a location-arc routing problem.

In mail delivery, the location of relay boxes along postman routes, studied by Bouliane and Laporte [4], is another application related to the SANRP. It consists of determining the location of relay boxes in such a way that when a postman runs out of mail, he goes to the relay box located on his route in order to replenish his bag. The location of these relay boxes is determined by considering the maximal weight a postman can carry, as well as the distance he travels to reach the relay boxes. These must be loaded in advance by a vehicle. When a postman needs to replenish his bag, he goes to the relay box which contains the mail he needs to deliver along the next segment of his route. A mail bag can be therefore viewed as a painting vehicle in the SANRP. However, no synchronization is required between the replenishment vehicle and the postman in this application.

3. Formal problem description

The SANRP is defined on a directed graph $G=(V,A'\cup A)$, where $V=\{0,\dots,n\}$ is the node set, node 0 represents the depot at which all vehicle routes start and end, A is the set of required arcs (street segments that must be painted), and A' is the set of non-required arcs (street segments that do not have to be painted). All arcs $(i,j)\in A$ must be serviced once in the solution, and any arc $(i,j)\in A'\cup A$ can be *deadheaded* any number of times, i.e., it can be traversed without being serviced. Every arc $(i,j)\in A$ has a non-negative demand p_{ij} which represents the amount of paint required to mark the street segment from i to j. Two kinds of vehicles are available: a replenishment vehicle r_v and a set R of homogeneous vehicles called painting vehicles. Each painting vehicle $r\in R$ has a tank of capacity Q which most of the time is not sufficient to paint the roads assigned to it. Therefore each painting vehicle requires a single or multiple refillings to perform its work. The replenishment vehicle r_v is used to supply paint to the painting vehicles, and we assume its capacity is sufficient to satisfy the total demand of the painting vehicles. Each painting vehicle $r\in R$ travels at speed s (km/h) when it is not painting a street segment, and at speed $s'<s$ when it is painting. Therefore, each arc $(i,j)\in A\cup A'$ has two associated travel times called t_{ij} and t'_{ij} for service and traversal, respectively. Initially, all painting vehicles leave the depot with a full tank, and every node on their route is considered as a potential refilling point. Replenishment can be made preventively before the tanks of the painting vehicles become empty. The refill nodes must be chosen so that the painting vehicles and the replenishment vehicle can meet at the same time without incurring too much waiting. A painting vehicle cannot start servicing an arc if it does not have enough paint for it. The amount of paint required between any two consecutive refill nodes should not exceed Q.

The goal of the SANRP is to determine a set of feasible and synchronized routes minimizing the *makespan*, i.e., the duration of the longest route. This helps generate balanced routes, which are viewed as a desirable feature in many contexts.

Amaya et al. [1] show that even in the simple case, when there is only one painting vehicle and one replenishment vehicle, the problem is NP-hard and the size and complexity of the related formulation are substantial. Our problem is even more difficult because there are several vehicles, preventive refills, and synchronization constraints. Thus formulating it would only be a mathematical exercise and no further insight into our understanding of the problem, and could not be used to solve even relatively small instances. Therefore, the SANRP can now be summarized as follows:

Objective: Minimize the duration of the longest route (painting vehicles).

Constraints:

- The painting vehicles should not run out of paint.
- The refill nodes must be determined together with the routes themselves.
- Preventive refills are allowed.