Computers & Operations Research 40 (2013) 1842–1851

Contents lists available at SciVerse ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

# Inventory management with log-normal demand per unit time

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ARTICLE INFO

Available online 4 February 2013

*Keywords:*
Continuous review
Demand during lead time
Inventory
Log-normal distribution
Mixtures of truncated exponentials

ABSTRACT

This paper examines optimal policies in a continuous review inventory management system when demand in each time period follows a log-normal distribution. In this scenario, the distribution for demand during the entire lead time period has no known form. The proposed procedure uses the Fenton-Wilkinson method to estimate the parameters for a single log-normal distribution that approximates the probability density function (PDF) for lead time demand, conditional on a specific lead time. Once these parameters are determined, a mixture of truncated exponentials (MTE) function that approximates the lead time demand distribution is constructed. The objective is to include the log-normal distribution in a robust decision support system where the PDF that best fits the historical period demand data is used to construct the lead time demand distribution. Experimental results indicate that when the log-normal distribution is the best fit, the model presented in this paper reduces expected inventory costs by improving optimal policies, as compared to other potential approximations.



## 1. Introduction

The problem of characterizing uncertain demand during lead time in continuous review inventory management models has been an ongoing subject of research for some time. In many approaches suggested in the literature, uncertain demand per unit of time (a day, for instance) and random lead time are modeled with standard probability density functions (PDFs). Probability distributions are assigned for demand per unit of time (DPUT) and lead time (LT), and a resulting distribution is assigned for demand during lead time, or lead time demand (LTD). The latter distribution is used to determine the inventory order quantity and the best time to place the order, and may be used to estimate other quantities such as expected stockout costs.

### 1.1. Lead time demand assumptions

The LTD distribution in a single-product, continuous review inventory system where both DPUT and LT are (possibly) random can be defined as follows. Suppose $X_\ell$, in each period $\ell = 1,\ldots,L$ are the independent and identically distributed (i.i.d.) random variables representing customer DPUT where $L$ is a random variable representing LT. This means that total customer demand over the $L$ periods, $X$, is determined as the following sum of i.i.d. random variables:

$$X = X_1 + X_2 + X_3 + \cdots + X_L. \quad (1)$$

The PDF $f_X$ for $X$ is often referred to as a compound probability distribution. Modeling the distribution of $X$ accurately can improve the selection of an order quantity ($Q$) and reorder point ($R$). In some cases, the DPUT component of LTD can be derived from separate random variables for order size and order intensity (or frequency) [1,2].

### 1.2. Previous research

The normal distribution is often used to model demand because of its convenient mathematical properties. However, in practice actual demand from customers for some products may be better modeled with an asymmetric, or skewed, probability distribution. The log-normal PDF is an example of a skewed probability distribution and appears as the function with the long right tail in Fig. 1, which also displays a familiar bell-shaped normal distribution. The focus of this paper is determining optimal inventory order quantity and reorder point policies when DPUT follows a log-normal PDF.

The goal of improving on the accuracy of the normal approximation to the LTD distribution led to the development of numerous models that incorporate other distributional assumptions. Some of these models are summarized in Table 1. The table identifies the distribution assumed (where applicable) for the

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http://dx.doi.org/10.1016/j.cor.2013.01.017

input variable and the distribution then constructed for LTD, which is often an approximation of the actual distribution. The value provided by the model to aid in establishing inventory policies is shown in the "Result" column. This table serves to update similar information provided by Bagchi et al. [3].

Bagchi et al. [1] give a general framework for establishing an LTD distribution by classifying LT as symmetric or skewed and DPUT on a continuum between slow moving and fast moving items. Based on the classification, gamma, normal, and Poisson distributions are assigned for LT and DPUT, and these lead to reasonable approximations for LTD. Mitchell et al. [13] assign a geometric distribution for DPUT and a Poisson distribution for LT. This model proved to be a good fit for inventory items maintained by the Air Force. Ord and Bagchi [10] suggest using a gamma approximation to the distribution for LTD when DPUT is normal and LT has a gamma distribution. Burgin [5] earlier recommended using a gamma distribution to approximate LTD due to its desirable statistical properties.

Fortuin [19] compares five different standard PDFs as DPUT models and notes that when LT is constant, the models often lead to similar results. Bagchi et al. [3] find, however, that when LT is also variable, using a close approximation to the actual distribution for LTD is important when calculating appropriate safety stock levels.

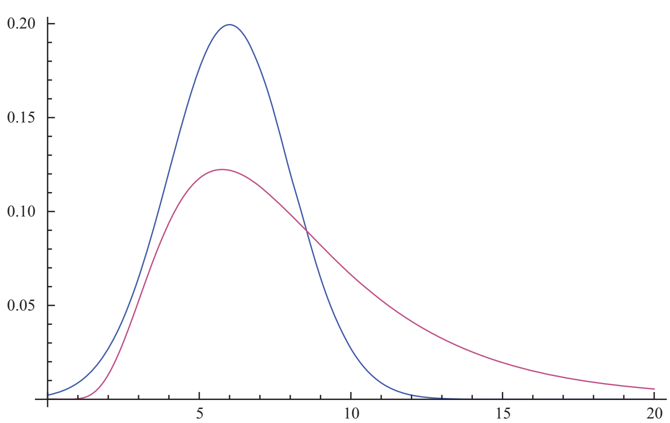


**Fig. 1.** Normal and log-normal probability distributions for demand per day.

Many of the approaches included in Table 1 involve finding an LTD distribution that is the best approximation, given the distributions assigned to order size, order intensity, DPUT, and/or LT. This distribution is often used to find stockout probabilities in order to set an order quantity and reorder point in a continuous review inventory system that will achieve a desired service level. A few find analytical expressions for approximately optimal values for order quantity and/or reorder points in terms of the parameters of the input distributions or the model assigned to LTD. Some approaches assume a periodic review inventory system is in place and establish heuristic approaches to determining whether an order is needed on a specific review date and possibly recommend an order size [14,15,17].

### 1.3. Log-normal demand

Previous research suggests that the log-normal distribution as a model for demand is of interest in the inventory management field. Das [12] introduces a method for finding the optimal inventory order quantity and reorder point under log-normal demand. This method is applicable when the distribution for demand over the entire LT period is log-normal, which is a scenario also studied by Tadikamalla [7]. Kamath and Pakkala [20] suggest that the log-normal distribution is a good model for demand in demonstrating a Bayesian approach to setting inventory policies in a fixed review model, and Dohi and Kaio [21] consider the case of lognormal demand in a production planning model. Brown [22] presents empirical evidence that demand for several types of products can be well-represented by the log-normal distribution. Crouch and Oglesby [23] incorporate a log-normal assumption to find expected demand for an inventory lot-size model.

We consider the specific case where the continuous PDF that most closely fits historical DPUT data is the log-normal

**Table 1**
Previous research on lead time demand distributions.

| Reference | Order size | Order intensity | Demand per unit time | Lead time | Lead time demand | Result |
|---|---|---|---|---|---|---|
| Taylor [4] | N/A | N/A | Poisson | Gamma | Negative binomial | $P(X>R)$ |
| Burgin [5] | N/A | N/A | Gamma | Constant | Gamma | $P(X>R)$ |
| Das [6] | N/A | N/A | Normal | Exponential | Exponential | $(Q^*,R^*)$ |
| Das [6] | N/A | N/A | Exponential | Geometric | Exponential | $(Q^*,R^*)$ |
| Tadikamalla [7] | N/A | N/A | N/A | N/A | Log-normal | $P(X>R)$ |
| Carlson [8] | N/A | N/A | Poisson | Exponential | Geometric | $(Q^*,R^*)$ |
| Nahmias and Demmy [9] | Logarithmic | Poisson | N/A | Gamma | Logarithmic-Poisson-gamma | $f_X$ |
| Ord and Bagchi [10] | N/A | N/A | Normal | Gamma | Gamma | $f_X$, $R^*$ |
| Bagchi et al. [11] | N/A | N/A | Poisson | Normal | Hermite | $P(X>R)$, R |
| Das [12] | N/A | N/A | N/A | N/A | Log-normal | $(Q^*,R^*)$ |
| Mitchell [13] | Geometric | Poisson | N/A | Constant | Compound Poisson | $P(X>R)$ |
| Mitchell [13] | Constant | Poisson | N/A | Constant | Compound Poisson | $P(X>R)$ |
| Segerstedt [14] | Gamma | Gamma | N/A | Gamma | N/A | $P(X>R)$ |
| Krever et al. [2] | Gamma or normal | Poisson | N/A | Constant | Normal | $P(X>R)$ |
| Larsen and Thorstenson [15] | Negative binomial | Erlang | N/A | Constant | Compound renewal | Fill rate |
| Larsen and Thorstenson [15] | Binomial | Erlang | N/A | Constant | Compound renewal | Fill rate |
| Larsen and Thorstenson [15] | Poisson | Erlang | N/A | Constant | Compound renewal | Fill rate |
| Broekmeulen and van Donselaar [16] | N/A | N/A | Gamma | Constant | Gamma | Optimal safety stock |
| Teunter et al. [17] | Gamma | Binomial | Compound N/A | Constant | N/A | Fill rate |
| Cobb [18] | N/A | N/A | Normal, gamma, or exponential | Any discrete or continuous | Mixture distribution | $f_X$ (used to find $(Q^*,R^*)$) |