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A robust transportation signal control problem accounting for traffic dynamics

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This article is dedicated in honor of Professor Robert Paaswell on his 70th birthday for his outstanding contributions to transportation research and leadership

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ABSTRACT

Transportation system analysis must rely on predictions of the future that, by their very nature, contain substantial uncertainty. Future demand, demographics, and network capacities are only a few of the parameters that must be accounted for in both the planning and every day operations of transportation networks. While many repercussions of uncertainty exist, a primary concern in traffic operations is to develop efficient traffic signal designs that satisfy certain measures of short term future system performance while accounting for the different possible realizations of traffic state. As a result,uncertainty has to be incorporated in the design of traffic signal systems. Current dynamic traffic equilibrium models accounting for signal design, however, are not suitable for quantifying network performance over the range of possible scenarios and in analyzing the robust performance of the system. The purpose of this paper is to propose a new approach—robust system optimal signal control model; a supply-side within day operational transportation model where future transportation demand is assumed to be uncertain. A robust dynamic system optimal model with an embedded cell transmission model is formulated. Numerical analysis are performed on a test network to illustrate the benefits of accounting for uncertainty and robustness.

1. Introduction

Transportation system analysis has been traditionally concerned with supply and demand analysis in some nominally ‘typical’ conditions. However, since traffic volume and capacities continue to change both within day and day to day, dynamic models accounting for time varying conditions have recently been developed. In addition, the rapid advances in real-time sensors and information technology have provided us the ability to capture the inherent uncertainty and opportunities to apply robust optimization approaches to transportation problems. These opportunities have begun to challenge the idea of planning for ‘nominal’ conditions. While the initial impetus has been realized in the context of natural disasters—such as earthquakes [3,15]—affecting the ‘connectivity’ of a road network, recent thinking has focused on broadening this definition to both planning and operational decision making [29,30,5].

The main focus of this work is to develop a robust dynamic signal optimization formulation that integrates both dynamic traffic assignment and signal control. Because the transportation network performance depends on the control devices such as traffic lights, variable message signs, etc., and influences the ‘optimal path’ available for routing, an integrated model is highly beneficial to optimize the entire transportation network. In addition, there is also uncertainty in the total number of vehicles that will move from a given origin to a destination pair in a time interval. Estimates of how much of demand will move is forecasted for each O–D pair, but these estimates are at best educated guesses. Based on the realization of the demand, the signal control will vary at each intersection and the optimal routing pattern for vehicles would appropriately be different. In addition, the design of signal control should be *resilient* for all realizations of demand. Such resilient control mechanisms will improve the overall transportation network performance. Most of the past work account for this by using static user/stochastic user equilibrium models. These models are however not applicable because: (1) they are not applicable for real-time management; (2) they do not account for traffic dynamics and (3) they do not account for uncertainty and robustness.

The core question addressed in this analysis is: How should the traffic signal control settings be designed optimally over time to best meet the requirements of the transportation network performance, given uncertainties in point to point demand over time and the need to account for robustness, recognizing that there are constraints that limit the cycle times, green times at intersections and restrictions in capacities and jam densities on the road network?

To address this core question, we develop a robust optimization model of the dynamic system optimal signal control problem that focuses on developing optimal signal times at intersections. To

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correctly reflect the uncertainties in the O–D demand, and the resulting implications on network performance, the optimization must include uncertainty in the structure of some of its constraints. In addition to accounting for robustness, the objective function should measure the resiliency of the network. We have included this 'structural uncertainty' in the definition of discrete scenarios and develop a discrete two stage robust formulation. We are focused more on finding 'robust' solutions to the optimization problem, using the core concepts discussed in [30,23]. This robust optimization formulation allows us to study the tradeoff of different objectives with varying levels of system risk in the solutions that are accepted by the network manager.

Although, the main focus of this work is demonstrating the need to account for robustness in traffic signal control on simple transportation networks, the developed model can be extended to solve large scale networks by using efficient algorithms for the robust optimization model. The paper is organized as follows. Section 2 discusses the previous literature on robust optimization and applications in transportation problems. Section 3 describes the formulation of the robust signal control problem and discusses the intuition of the objective function and the constraint sets. Computational results on a test network are presented in Section 4. Section 5 focuses on discussing the insights obtained from the analysis and Section 6 concludes the paper with recommendations for future work.

2. Background

A common tool for modeling uncertainty is a two-stage stochastic program where the decision variables are partitioned into two sets. First stage variables are those that have to be decided before the uncertain parameters are realized. For transportation planning, these would be equivalent to any infrastructure added to the network before future demand is realized. The second stage (also referred to as recourse) variables represent decisions that are made after some uncertainty has been realized. The second stage problem can be viewed as an operational decision making problem following the first stage plan. It is important to realize that the second stage objective is to minimize the expected value of a function of the random second stage costs. This concept has been applied in linear, integer and non-linear programming problems. A compact formulation of a general stochastic linear programming is given in [13,7].

Waller and Ziliaskopoulos [31] proposed a dynamic network design model as a two stage stochastic linear programming problem where the cell transmission model (CTM) [10] is the embedded traffic flow model in the second stage and the demand was modeled as a random variable. Lo and Tung [21] discuss a chance-constrained reliability formulation of the traffic equilibrium problem under minor network disruptions. Their primary focus was on developing a probabilistic user equilibrium model under the assumption that users minimize the expected travel time and the flows would settle into equilibrium in the long term. In these formulations, minimizing expected costs often fails to appropriately account for extreme outcomes which are resilient to future variations. In other words, although the first stage variables will optimize the mean of the objective function, there may be scenarios for which the network performs poorly although on average it performs quite satisfactorily.

Long-term demand uncertainty can be accounted for using stochastic optimization methods with either a recourse or a chance-constrained formulation as demonstrated in [31]. In transportation, uncertainty has been primarily studied in terms of capacity reliability of a network [12]. 'Capacity reliability' has been defined in different ways by different researchers. A comprehensive review of these definitions is given in [2]. Chen et al. [8] defined capacity reliability as the probability that the network can accommodate a certain demand at a given service level. The studies were done on small networks, extending these studies for larger networks is a computationally intensive task. Du and Nicholson [11] proposed a conventional equilibrium approach with variable demand to describe flows in a network with degradable link capacities. Lo and Tung [22] define capacity reliability as a maximum flow that the network can carry, subject to link capacity and travel time reliability constraints.

The problem of robustness has been closely examined in the area of financial investment and is often addressed by including the variance of future cost as a measure for analysis (for example, refer to [23]). This approach minimizes variance as part of the objective function so that highly volatile solutions are discouraged. An efficient frontier is achieved by varying the weights on the expected value and volatility of network performance in the objective function. This approach is appropriate if input parameters are uncertain with known distributions, or if there exist multiple bounded random input parameters with unknown distributions. The model presented in [23] extends the volatility to higher norms of the random variables and has been extensively used in practice [24]. One drawback, however, is that it requires symmetrically distributed random variables. A second approach is based on the von Neumann–Morganstern expected utility models [14]. This presents a more general framework for handling risk aversion, the primary advantage being the ability to handle asymmetries in random variables. These models can also be extended to model multi-period planning problems. In our work we use this definition of robustness to model the minimization of network wide travel times.

A more recent definition of robustness is given in [6]. A robust solution is defined at an aggregate level as one that guarantees the feasibility of the solution if, for a given number i, less than i constraint coefficients change. Further, a probabilistic guarantee that the robust solution will be feasible is given if more than i coefficients change. Ben-Tal and Nemirovski [4] proposed a second-order cone programming approach to overcome the conservative solutions of Soyster [28]. The formulation is a non-linear program and has a difficult solution algorithm in the size of constraints and variables. This definition of robustness however converts our problem into the worst case problem [28].

3. Model formulation

In this section, we define the basic variables and the mathematical formulation for developing the robust signal control problem. The formulation utilizes an embedded cell transmission model [9,10], a mesoscopic traffic flow model to capture the vehicle movement in the network. CTM [9,10] provides a convergent numerical approximation to Lighthill and Whitham [16] and Richards' [25] (LWR) hydrodynamic model to simple difference constraints by assuming a piecewise linear relationship between traffic flow and density for each cell (or segment). The CTM approximates the fundamental diagram of flow-density shown in Fig. 1a by a piecewise linear model shown in Fig. 1b. The basic relationships of the cell transmission model are extensively discussed in [9,10,20,18,32]. To facilitate cross-reference, we adopt similar notation as in [32,29]. The CTM as proposed by Daganzo [9,10] does not explicitly model signalized intersections; however, the same basic building blocks can be extended to capture traffic realism. Beard and Ziliaskopoulos [1] develop an improvised CTM that can explicitly model intersection movements not accounting for the demand uncertainty. The intersection cell configuration adopted in this paper is similar to the one by Beard and Ziliaskopoulos [1] and is briefly described here. Each turning movement on each approach at the intersection is designated a separate single cell. Each cell uniquely handles a turning movement. The set of cells that represent all the movements at an intersection are together