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Region-rejection based heuristics for the capacitated multi-source Weber problem

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ABSTRACT

A new type of constructive and adaptive heuristics is put forward to generate initial solutions for the capacitated multi-source Weber problem. This technique is based on guiding the search by constructing restricted regions that forbid new locations to be sited too close to the previously found locations. In this work, a restricted region is represented by a circle whose radius is initially set to a fixed value, based on the sparsity of the customers and the number of facilities, and then a scheme that dynamically adjusts the radius at each facility is proposed. A discretisation technique that divides a continuous space into a discrete number of cells while embedding the use of restricted regions within the search is also presented. The experiments show that the proposed region-rejection methods, though simple and easy to understand, provide encouraging results with regard to both solution quality and computational effort. Some future research avenues are also briefly highlighted.

1. Introduction

In this study, we deal with a problem where we are given a set of customers, located at n fixed points, with their respective demands. We are required to locate M facilities in continuous space to serve these n customers, and to find the allocation of these customers to these M facilities without violating the capacity of any of the facilities. The objective is to minimize the sum of the weighted Euclidean distances. This capacitated continuous location–allocation problem is also known as the capacitated multi-source Weber problem (CMSWP) which can be formulated as follows:

$$\text{Minimize} \quad \sum_{i=1}^{M}\sum_{j=1}^{n} x_{ij} d(X_i, a_j) \tag{1}$$

Subject to

$$\sum_{i=1}^{M} x_{ij} = w_j, \quad j = 1, \ldots, n \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} \leqslant b, \quad i = 1, \ldots, M \tag{3}$$

$$x_{ij} \geqslant 0, \quad i = 1, \ldots, M; \; j = 1, \ldots, n \tag{4}$$

where M is the number of facilities to be located; x_{ij} is the quantity assigned from facility i to customer j, $i = 1, \ldots, M$; $j = 1, \ldots, n$; $d(X_i, a_j)$ the Euclidean distance between facility i and customer j; $a_j = (a_j^1, a_j^2) \in \Re^2$ is the location of customer j; $X_i = (X_i^1, X_i^2) \in \Re^2$ are coordinates of facility i; w_j the demand, or weight, of customer j, where $w_j \in N$; and b a fixed capacity of a facility, where $b \in N$.

The objective function (1) is to minimize the sum of the transportation costs. Constraints (2) guarantee that the total demand of each customer is satisfied. Constraints (3) ensure that capacity constraints of the facilities are not exceeded, and constraints (4) refer to the non-negativity of the decision variables. Note that once the M facilities are located, the problem reduces to the classical transportation problem (TP).

The solution of this continuous location problem may obviously be infeasible in practice as the locations could be on the lakes, mountains, forests, etc. Nonetheless there are applications such as locating oil drills in the sea or desert, see e.g. Rosing [1]. For a more general review of applications-oriented literature, see Hodgson et al. [2]. Furthermore, a solution of this problem could be used as a benchmark for the discrete problem, or simply as a solution that can be transformed into a feasible one, for example by incorporating barriers into such location problems (see e.g. the recent work of Bischoff and Klamroth [3]).

This paper is organized as follows: in the next section we present a review of the relevant literature. The sections thereafter address the proposed heuristics. This is followed by a section on computational results. Finally, the last section presents our conclusions and highlights some research avenues that we believe are worthwhile exploring in the future.

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2. Literature review

Most of the literature on the continuous location problem focuses on the multi-source Weber problem (MSWP). Rosing [1] finds the location of steam generators in the Orinoco heavy oil belt of Venezuela by solving the MSWP. Hansen et al. [4] solve the MSWP through the p-median problem by considering all fixed points as potential facility sites. Brimberg et al. [5] carry out a comparison of heuristics including those based on variable neighbourhood search and genetic algorithms. Gamal and Salhi [6] present a constructive heuristic based on the furthest distance rule to find initial locations while introducing forbidden regions to avoid locations being too close to each others. Gamal and Salhi [7] create a discretisation based approach known as a cellular heuristic whereas Salhi and Gamal [8] adopt a genetic algorithm to solve the problem. Taillard [9] proposes a decomposition/recombination heuristic that partitions the problem into smaller subroblems, which are then solved by a candidate list search for various number of centres. Aras et al. [10] tackle the MSWP using vector quantization and self-organizing maps for both Euclidean and rectilinear distances. A variant of this problem is the constrained Weber problem which is also known as the Weber problem in the Presence of Forbidden Regions and/or Barriers to Travel. This is investigated by Katz and Cooper [11] and Hansen et al. [12]. For more details on this subject see Bischoff and Klamroth [3].

There is, however, a shortage of papers on the CMSWP. The earliest work in this area was conducted by Cooper [13] who develops exact and heuristic methods. In the exact method, the idea is to generate all basic feasible solutions using the TP. Starting with any feasible basic solution, the idea is to construct the connected graph of all basic feasible solutions. For every solution, the location problem is solved and the solution which yields the minimum cost is chosen as the optimal solution. In the heuristic method, the alternating transportation–location heuristic known for short as ATL was proposed. Fundamentally, ATL is a modification of the heuristic method originally developed by Cooper [14] for the pure location–allocation problem known as the alternate location–allocation method. The idea of ATL is that the location–allocation problem and the TP are alternately solved until no epsilon (ε) improvement in total cost is found. Cooper [15] modifies a heuristic method initially used for the fixed charge problems in Cooper [16]. It is only in 1994 that the problem was revisited. Sherali et al. [17] formulate the rectilinear distance CMSWP as a mixed integer nonlinear programming formulation, and proposed a reformulation–linearization technique (RLT) to transform the problem into a linear mixed-integer program. Gong et al. [18] put forward a hybrid evolutionary method based on a genetic algorithm to search the locatable area to find the global or near global locations. In the allocation stage, a Lagrange relaxation method was applied. Experiments were carried out on randomly generated data with the number of facilities (M) varying from 2 to 6. Sherali et al. [19] design a branch and bound approach based on a partitioning of the allocation space to develop global optimization procedures for the capacitated Euclidean and $\boldsymbol{l}_p$ distance MSWP. Two bounding schemes were also put forward based on solving a projected location space subproblem and a variant of RLT that reformulated the problem into a linear programming relaxation. Aras et al. [20] propose three heuristic methods that use Lagrangean heuristic, the discrete p-capacitated facility location problem which is similar to the p-median method of Hansen et al. [4], and the cellular heuristic of Gamal and Salhi [7] to deal with the CMSWP with Euclidean, squared Euclidean, and $\boldsymbol{l}_p$ distances. In a subsequent research, Aras et al. [21] adopt their earlier approaches to solve the CMSWP with rectilinear distance. Aras et al. [22] tackle the CMSWP with rectilinear, Euclidean, squared Euclidean, and $\boldsymbol{l}_p$ distances by adopting simulated annealing, threshold accepting, and genetic algorithms. These heuristics perform well when tested on the Sherali et al.'s data sets. Sherali et al. and Aras et al. used their approaches on small instances ($n \leqslant 30$) where the capacity of a facility is not necessarily the same. Zainuddin and Salhi [23] present a perturbation-based heuristic for solving the Euclidean CMSWP. A perturbation scheme was designed by considering borderline customers to form clusters. These customers are those that lie approximately half-way between their nearest and their second nearest facilities. Clusters of customers were constructed and taken out temporarily whilst the TP was performed, and then they were introduced back again when finding the new location. This heuristic outperformed the classical ATL when tested on large instances ($n = 50$–1060) with facilities having the same capacities.

The reader is referred to the recent comprehensive review on continuous location–allocation problems by Brimberg et al. [24].

3. Solution framework

Our overall solution method is based on Cooper's alternating transportation–location heuristic. initially, M open facilities are chosen from the customer points (fixed points), then the TP, using these M open facilities, is employed to obtain the allocation for this capacitated problem. Here, the output is the M independent set of allocations, each subset consisting of n_i fixed points where $i = 1, 2, \ldots, M$ and $\sum_{i=1}^{M} n_i \geqslant n$. Note that we used '$\geqslant$' instead of '$=$' as some customers may have their demand split between different facilities during the allocation stage. An iterative procedure based on the Weiszfeld equations, as given in Eq. (5), is then applied to find the new location of each ($i = 1, 2, \ldots, M$) of these M facilities.

$$X_i^{1(k)} = \frac{\sum_{j_i=1}^{n_i} w_{j_i} a_{j_i}^1 / d(X_i^{(k-1)}, a_{j_i})}{\sum_{j_i=1}^{n_i} w_{j_i} / d(X_i^{(k-1)}, a_{j_i})} \quad \text{and}$$
$$X_i^{2(k)} = \frac{\sum_{j_i=1}^{n_i} w_{j_i} a_{j_i}^2 / d(X_i^{(k-1)}, a_{j_i})}{\sum_{j_i=1}^{n_i} w_{j_i} / d(X_i^{(k-1)}, a_{j_i})} \tag{5}$$

where the superscript k designates the iteration number within the Weiszfeld iterative procedure; $(a_{j_i}^1, a_{j_i}^2)$ represents the location of the set of n_i fixed points, $j_i = 1, 2, \ldots, n_i$; $(X_i^{1(k)}, X_i^{2(k)})$ denotes the new location of the ith facility at iteration k ($i = 1, 2, \ldots, M$); and w_{j_i} corresponds to all or a fraction of the jth customer demand that is served by facility i.

In other words, the demand of some customers might have been split as a result of the solution of the TP (i.e., $w_{j_i} \leqslant w_j$). Therefore some customers may be utilized more than once in Eq. (5), but each time only with a portion w_{j_i} of their demand. The process of alternating between the location–allocation problem and the TP is then applied until no improvement in total cost can be found. Fig. 1, which is used as a basis for our research, shows the main steps of Cooper's alternating transportation–location heuristic. In Step 1, Cooper [13] selects randomly locations from customer locations as the initial facility configuration. In this study, we propose, instead in Step 1, simple but efficient ways of guiding the search for generating initial starting locations. The idea is to restrict generating initial locations near those areas that contain the already found locations. The way these restricted regions are constructed makes up the main contribution of this study and will be presented in subsequent sections. To increase the chance of reaching a near optimal solution, the method is reiterated several times, say K times, using different random starting locations.

4. A region-rejection based approach

In this section, we introduce the idea of restricted regions which forbid certain areas close to the previously found locations to be considered in future for the choice of the remaining open facilities.