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Covering a line segment with variable radius discs

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Abstract

The paper addresses the problem of locating sensors with a circular field of view so that a given line segment is under full surveillance, which is termed as the *disc covering problem on a line*. The cost of each sensor includes a fixed component *f*, and a variable component that is a convex function of the diameter of the field-of-view area. When only one type of sensor or, in general, one type of disc, is available, then a simple polynomial algorithm solves the problem. When there are different types of sensors, the problem becomes hard. A branch-and-bound algorithm as well as an efficient heuristic are developed for the special case in which the variable cost component of each sensor is proportional to the square of the measure of the field-of-view area. The heuristic very often obtains the optimal solution as shown in extensive computational testing.

Scope and purpose

Problems of locating facilities to cover sets of points on networks and planes have been widely studied. This paper focuses on a new covering problem that is motivated by an application where a line segment is to be kept under surveillance using different types of radars. Using reasonable assumptions, some nonlinear covering problems are formulated. Efficient exact algorithms and heuristics are developed and analyzed for "easy" and "hard" cases, respectively.

Keywords: Sensor location; Network covering problems; Mixed integer nonlinear programming

1. Introduction

In this paper we introduce and study a new locational decision problem: given a set of discs with variable radii with costs depending on their radii and fixed costs, find a subset covering a unit length segment at minimum cost.

This problem was motivated by the following application, part of which was an industry-funded radar surveillance project at The University of Arizona. We have a river over which we need to track possible activities of non-collaborative or antagonistic objects or people (e.g., unauthorized boats, dangerous floating objects, swimmers, etc). For this purpose, we need to locate a set of radars so that every point on the river is under surveillance by at least one radar. It is assumed that the river can be modeled as a tree network consisting of line segments and that each radar has a field of view defined by a radius and an angle of view (a pie-shaped coverage), with this angle large enough so that the coverage area may be

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approximated as a disc. Although the problem is relatively easily stated, the actual locational decision is complicated due to several additional factors. Coverage depends not only on the river topology, radar type and power, but also on several parameters such as width of river and obstacles over it, potentially forbidden areas where radars may not be located, elevation of the potential location sites, and other characteristics associated with the physical environment, dealing with, for example, the atmospheric and water conditions. Further details on this scenario and the scope of the project are reported in [1].

This radar sensors location model relates to several broad classes of geometric locational problems. Many important land-use planning decisions deal with locating facilities at sites, choosing from a given set of potential sites, so that another given set of points are "covered" (i.e., they are within a specified distance from the closest located facility) while optimizing a specified objective. Models for locating at points within continuous spaces, as well as locating among set of discrete points or on a network, are widely used by geographers, regional scientists, network planners, and others facing locational decisions problems which can be modeled as such covering problems (for a comprehensive review of the literature see, for example, [2–5]). From the methodological viewpoint, the radars location problem relates closely to the class of geometric covering problems where potential facilities and demand points are embedded on a Euclidean plane, for which there is considerable literature. We briefly review below results that are most relevant for our application.

Problems related to *covering with discs* consists of identifying the minimum number of discs with fixed radius to cover a given set of points in the plane. A number of articles have appeared in the last three decades addressing this NP-hard problem. In 1975, Chvátal introduced the *Art Gallery Problem* in [6], where one has to find the minimum number of watchmen (or cameras) needed to observe every wall of an art gallery room. The art gallery is assumed to be a n-sided polygon, possibly with polygonal holes. It has been shown that an art gallery with h holes and n walls (including holes' sides) requires at most $\lfloor (n+h)/3 \rfloor$ watchmen (the bound is tight, see [7,8]). Another important paper, by Hochbaum and Maas [9], presents polynomial approximation algorithms for different versions of geometric covering problems, including covering by discs. Subsequently, several papers have appeared with improved approximation factors and running times (see for example, [10,11]).

The problem of partial covering, also referred to as the robust k-center problem, is analyzed in [12], where computational complexity is discussed and approximation algorithms together with computational evidence of their performance are provided.

The geometric Disc Covering Problem relates also to the deployment of wireless transmission networks. Surveys on coverage problems dealing with this particular application can be found in [13,14]. We limit our literature review to a few papers dealing with applications similar to the radar sensors location problem. Alt et al. [15] consider a problem where a set of points demand connectivity. Their goal is to locate a set of sensors, modeled as discs with variable radii, covering the demand points at minimum total cost. Each sensor's transmission cost has the form $f(r) = r^{\alpha}$ where r is the covering radius of the sensor. Several results are presented in [15], including complexity characterization and approximation algorithms. Although different scenarios are addressed, depending on possible restrictions on discs' locations and demand points, their analysis is limited to discrete sets of points.

Article [16] addresses the problem of locating base stations for wireless communication where the demands and potential facilities are represented by a discrete set of points and each station can broadcast up to a maximum distance. A polynomial approximation scheme is given, together with complexity results. The following disc-covering geometric problem applied to wireless communication is addressed by Franceschetti et al. [17]: given an infinite square grid G, determine how many discs, centered at the vertices of G, with a given radius r, are required, in the worst case, to completely cover a disc with the same radius arbitrarily placed on the plane. The authors show that this number is an integer in $\{3, 4, 5, 6\}$ depending on r and on the grid spacing. In addition, they discuss the applicability of this model to the design of approximation algorithms for facility locations on regular grids and to base station placement for wireless communication. The expected quality of service (level of surveillance) of a given sensor network is analyzed in [18,19], where the authors exploit computational geometry and graph theoretic techniques, such as Voronoi diagrams, Delaunay triangulation and graph search, to design exact polynomial algorithms for some special cases.

Location of railway stops is another application of the Disc Covering Problem. In [20], the effect of introducing additional stops in the existing railway network is addressed. The problem comprises of covering a set of points in the plane by discs with the restriction that their centers have to lie on a set of line segments that represents the railway tracks. A similar problem is addressed in [21], where the discs must be centered on two intersecting lines.