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Locating a semi-obnoxious facility with expropriation

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Abstract

This paper considers the problem of locating semi-obnoxious facilities assuming that "too close" demand nodes can be expropriated by the developer at a given price. The objective is to maximize the minimum weighted distance from the facility to the non-expropriated demand nodes given a limited budget while taking into account the fact that customers do not want to be too far away from the facility. Two models of this problem on a network are presented. One is to minimize the difference between the maximum and the minimum weighted distances. The other one is to maximize the minimum weighted distance subject to an upper bound constraint on the maximum weighted distance. The dominating sets are determined and efficient algorithms are presented.

Keywords: Semi-obnoxious facility location; Expropriation; Minimax; Maximin

1. Introduction

During the past few decades, the literature for location models has been greatly expanded. Most of the models consider the problem of locating either desirable service facilities or obnoxious (undesirable) service facilities. For desirable facility location models, the objective is to minimize some function of distance between facilities and customers. Examples are minisum (minimize the weighted sum of distances) and minimax (minimize the weighted distance between the facilities and the farthest customer). Refs. [1–7] give a general overview of models on locating desirable facilities. For obnoxious facility location models, the objective is to maximize some function of distance. A typical objective is maximin (maximize the weighted distance between the facilities and the closest customer). For more details on obnoxious facility location models, interested readers are referred to [8–12].

In reality, however, some facilities cannot be strictly categorized as desirable or obnoxious. For example, consider the problem of locating an airport. On the one hand, customers would like the airport to be close so that they do not need to travel a long distance to receive service. On the other hand, customers do not want the airport to be too close because it generates noise and pollution. Such facilities are called semi-obnoxious facilities.

It is obvious that the traditional *minimax* criterion for desirable facilities is not appropriate for semi-obnoxious facilities because customers too close to the facilities are ignored. The traditional *maximin* criterion for obnoxious facilities is not appropriate for them either because the resulting location might be too far away from some customers. In order to address this conflict, some researchers utilized bicriteria methodologies. For examples, Brimberg and Juel

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[13] explored the efficient frontier for two objectives: minimizing the weighted sum of distances and minimizing the weighted sum of Euclidean distances raised to a negative power. Melachrinoudis [14] considered the minisum and maximin criteria using rectilinear distances.

This paper takes into consideration that an expropriation budget is available to compensate customers that are too close to the facility. The objective is to locate the semi-obnoxious facility so that the facility is sufficiently far from the closest non-expropriated customer and sufficiently close to the farthest customer.

Berman et al. [15] introduced the expropriation location problem with two models for locating a single obnoxious facility. The first one which is an extension of the obnoxious facility location problem is to maximize the minimum distance from the facility to the non-expropriated demand nodes subject to a given expropriation budget. The second one is to minimize the expropriation cost while ensuring that the facility is not within a given distance from the non-expropriated demand nodes. This model is related to the maximum cover models (see [16,17]).

In Berman and Wang [18], an expropriation model for semi-obnoxious facilities was presented. The objective is to locate the facilities so as to minimize the sum of weighted transportation cost and expropriation cost, assuming that demand nodes within a certain distance from an open facility are expropriated at a given price.

The rest of the paper is organized as follows. Some notation is introduced in the next section. Section 3 focuses on the model of minimizing the difference between the maximum and the minimum weighted distance while Section 4 is devoted to the model of maximizing the weighted minimum distance subject to an upper bound constraint on the maximum weighted distance. Finally, the paper is concluded with a summary.

2. Notation

Consider a connected network $G=(N,A)$ where $N=\{1,\ldots,n\}$ is the set of nodes and A is the set of links with $m=|A|$. The demands are located at the nodes of the network. Each demand node i has two associated parameters: weight $w_i \geqslant 0$ and expropriation cost $c_i > 0$. The weights can be based for example on the importance of a node (e.g. population size) where more important nodes receive smaller weight. Therefore, if there are two equally close demands and one is more important than the other, the demand which is more important will have a greater effect on the solution of the maximin problem. A budget B is given for expropriation. For any two points $x, y \in G$, $d(x,y)$ is the shortest distance from x to y. For any point $x \in G$, $w_i d(i,x)$ are sorted in non-decreasing order and we denote the sorted indices as $i_1^x, i_2^x, \ldots, i_n^x$. Let E_x be the set of demand nodes expropriated when the facility is located at x, then $E_x=\{i_1^x,\ldots,i_p^x\}$ where $\sum_{j=1}^{p} c_{i_j^x} \leqslant B$ and $\sum_{j=1}^{p+1} c_{i_j^x} > B$. For a facility located at x, let $f(x)$ and $g(x)$ be the minimum and maximum weighted distance from the facility to the non-expropriated demand nodes, respectively, i.e.,

$$g(x)=\min_{i\notin E_x} w_i d(i,x),$$
$$f(x)=\max_{i\notin E_x} w_i d(i,x).$$

Let $i^g(x)$ and $i^f(x)$ be the non-expropriated demand nodes that attain the minimum and maximum value of $w_i d(i,x)$, respectively, i.e., $g(x)=w_{i^g(x)} d(i^g(x),x)$ and $f(x)=w_{i^f(x)} d(i^f(x),x)$.

From the definition of E_x, we can see that E_x may not be uniquely defined for a given x. However, no matter whether or not E_x is unique, $g(x)$ and $f(x)$ are uniquely defined. For example, consider the network in Fig. 1. Suppose $B=2$, $w_i=1$ and $c_i=1$, $\forall i \in N$. The distance functions $d(i,x)$ for x on link (2,3), $\forall i \in N$ are shown in Fig. 2 (here x denote a point on link (2,3) at distance x from node 2). $f(x)$ is the upper envelop of all the distance functions, which is the upper bold lines. Since $c_i=1$ and $B=2$, at any point x, E_x consists of two nodes. $g(x)$ is shown by the lower bold lines. We can see that at point $x=1$, E_x can either be $\{1,2\}$ or $\{2,3\}$. Although E_x is not uniquely defined at this point, $g(1)$ is uniquely defined with a value of 4.

If $B=0$, we have $E_x=\emptyset$, $\forall x \in G$. Hence, $g(x)$ is the objective function of locating an obnoxious facility. It is piecewise linear concave on a link with $O(n)$ breakpoints. Furthermore, it can be computed in $O(n\log n)$ time by using standard techniques from computational geometry such as divide and conquer approach (see [19]). However, when $E_x \neq \emptyset$, $g(x)$ may no longer be concave on a link.

The next two sections present two models and their corresponding solution procedures.