Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Computers & Operations Research 35 (2008) 212–225
computers & operations research
www.elsevier.com/locate/cor

# Heuristic and exact algorithms for generating homogenous constrained three-staged cutting patterns

Yaodong Cui*

*Department of Computer Science, Guangxi Normal University, Guilin 541004, China*

Available online 3 April 2006

**Abstract**

An approach is proposed for generating homogenous three-staged cutting patterns for the constrained two-dimensional guillotine-cutting problems of rectangles. It is based on branch-and-bound procedure combined with dynamic programming techniques. The stock plate is divided into segments. Each segment consists of strips with the same direction. Only homogenous strips are allowed, each of which contains rectangles of the same size. The approach uses a tree-search procedure. It starts from an initial lower bound, implicitly generates all possible segments through the builds of strips, and constructs all possible patterns through the builds of segments. Tighter bounds are established to discard non-promising segments and patterns. Both heuristic and exact algorithms are proposed. The computational results indicate that the algorithms are capable of dealing with problems of larger scale. Finally, the solution to a cutting problem taken from a factory that makes passenger cars is given.


*Keywords:* Cutting; Packing; Cutting stock; Constrained two-dimensional cutting

## 1. Introduction

The constrained two-dimensional cutting (CTDC) problem discussed is as follows: $m$ types of rectangles are to be cut from a rectangular plate with size $L \times W$, where any cuts that are made are restricted to be guillotine cuts. The $i$th type has size $l_i \times w_i$, value $c_i$ and demand $d_i$, $i = 1, \ldots, m$. Assume that pattern $A$ includes $z_i$ pieces of type $i$. Assume that $\boldsymbol{N}$ is the set of natural numbers. The mathematical model for the CTDC problem is

$$\max\left(\sum_{i=1}^{m} c_i z_i\right), \quad A \text{ is a feasible pattern}, \quad z_i \in \boldsymbol{N} \text{ and } 0 \leqslant z_i \leqslant d_i, \quad i = 1, \ldots, m.$$

The related unconstrained two-dimensional cutting problem (UTDC) is

$$\max\left(\sum_{i=1}^{m} c_i z_i\right), \quad A \text{ is a feasible pattern}, \quad z_i \in \boldsymbol{N}, \quad i = 1, \ldots, m.$$

* Tel.: +86 773 5851286.
*E-mail address:* ydcui@263.net.

0305-0548/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cor.2006.02.029

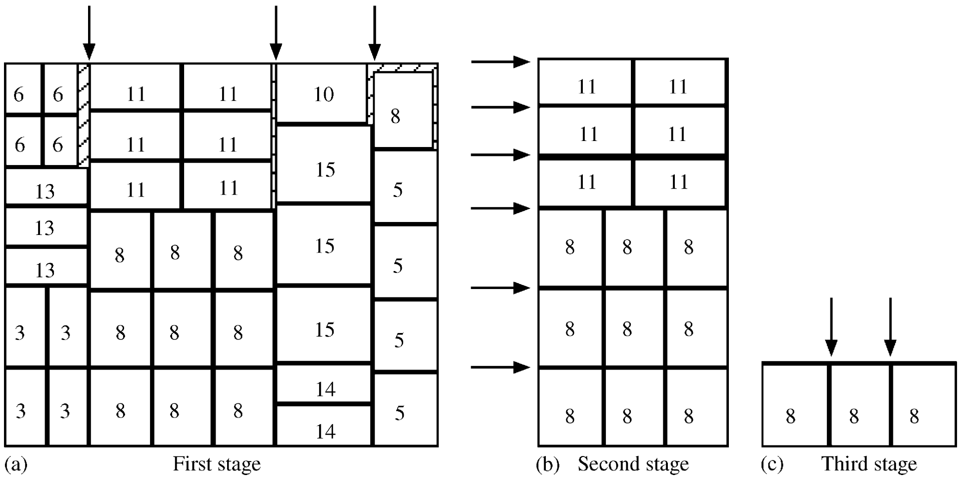


Fig. 1. Three-staged $X$-pattern. (a) First stage. (b) Second stage. (c) Third stage.

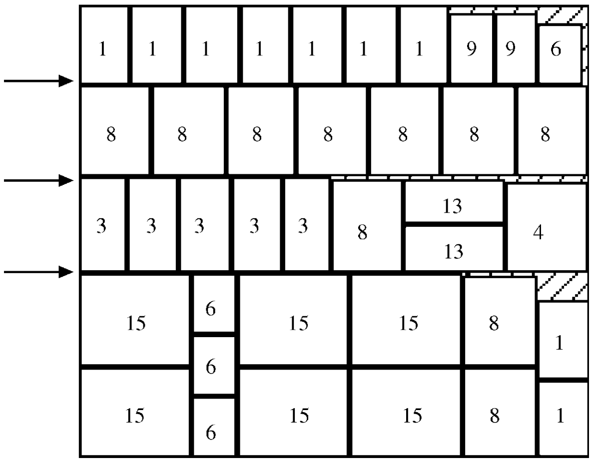

Fig. 2. Three-staged $Y$-pattern.

Both problems are denoted as the two-dimensional, rectangular SLOPP type, where SLOPP means single large object placement problem [1]. They appear in several industrial areas, such as the cutting of metal plate, wood plate, or plate glass into smaller rectangles. Good algorithms for these problems are important for the improvement of material usage.

This paper proposes the homogenous three-staged patterns shown in Figs. 1 and 2, where the numbers indicate the piece types. A three-staged pattern can be divided into pieces in three stages. For the pattern in Fig. 1, vertical cuts divide the plate into four segments at the first stage (Fig. 1a), horizontal cuts divide the segments into strips at the second stage (Fig. 1b), and vertical cuts divide the strips into pieces at the third stage (Fig. 1c). A pattern is referred to as a homogenous one if only pieces of the same type can appear in a strip; otherwise it is a general one. The pattern in Fig. 1a is an $X$-pattern, where the segments are arranged horizontally from left to right. Whereas the pattern in Fig. 2 is a $Y$-pattern, where the segments are arranged vertically from bottom to top.

Sometimes the shearing and punching process is used to divide the plate into pieces [2,3]. It includes two stages. At the first stage, a guillotine shear cuts the plate into strips. At the second stage, a stamping press punches out the pieces from the strips. Typically, a strip is fed into a stamping press that cuts a piece from the strip with each stroke, moves the strip forward, and cuts the next piece. The feeding distance between two adjacent pieces is referred to as a step. Fig. 3 shows two strips that may be processed at the second stage. The approach of this paper can directly deal with the cutting of a plate into strips with equal steps (Fig. 3a). In generating cutting patterns, the strip in Fig. 3b can be