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A comprehensive and robust procedure for obtaining the nofit polygon using Minkowski sums

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Abstract

The nofit polygon is a powerful and effective tool for handling the geometric requirements of solution approaches to irregular cutting and packing problems. Although the concept was first described in 1966, it was not until the early 90s that the general trend of research moved away from direct trigonometry to favour the nofit polygon. Since then, the ability to calculate the nofit polygon has practically become a pre-requisite for researching irregular packing problems. However, realization of this concept in the form of a robust algorithm is a highly challenging task with few instructive approaches published. In this paper, a procedure using the mathematical concept of Minkowski sums for the calculation of the nofit polygon is presented. The described procedure is more robust than other approaches using Minkowski sum knowledge and includes details of the removal of internal edges to find holes, slits and lock and key positions. The procedure is tested on benchmark data sets and gives examples of complicated cases.

Scope and purpose Cutting and packing problems involving irregular shapes feature in a wide variety of manufacturing processes. Automated solution techniques that can generate packing arrangements more efficiently than current technology that employs user intervention, must be able to handle the complex geometry that arises from these problems. The nofit polygon has been demonstrated to be an effective tool in providing efficient handling of the geometric characteristics of these problems. The paper presents a new algorithmic procedure for deriving this tool.

Keywords: Cutting and packing; Nesting; Geometric algorithms; Configuration space

1. Introduction

Cutting and packing problems involving irregular shapes are common in a variety of manufacturing processes. They occur whenever a piece of irregular shape is to be cut from a sheet of stock material. Examples include dye-cutting in the engineering sector, parts nesting for shipbuilding, marker layout in the garment industry, and leather cutting for shoes, furniture and other goods. The paper specifically addresses the geometric calculations required for tackling these problems. Here we consider that shapes are irregular if they are; polygonal, i.e no arcs; simple, i.e. non-self-intersecting; and non-rectangular. Even when all the components are rectangular the problem of finding layouts that minimize waste is known to be NP-hard. Where irregular components are involved an extra dimension of complexity is generated by the geometry.

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The precise requirements of a good layout will differ from industry to industry and this has led to a variety of algorithmic approaches. In spite of their differences, all the methods have a common requirement in which they need to be able to identify whether a layout is feasible or not, i.e. do any of the pieces overlap. Early research handled this problem in a number of ways. Adamowicz and Albano [1] chose to nest pieces into simpler shapes where the geometry can be more easily calculated. If the shapes are used directly then the intersection of pieces can be handled by direct trigonometric approaches such as the D function [2,3]. Alternatively the stock sheet and the pieces can be approximated as grid squares, often referred to as the raster method. Hence, if a piece occupies, fully or partially, a grid square it is coded as occupied [4,5].

Although all these approaches have merit, it is widely recognized that the nofit polygon (NFP) is more efficient, provided you have a robust and efficient NFP generator, and has become the principle approach for handling the geometry in nesting problems. Unfortunately, some researchers believe that despite the value of this tool, its introduction may have stifled research into this variant of packing problems. Wäscher et al. [6] report that there have been only 21 publications in irregular problems in the last 10 years. Researchers attribute this to the fact that the realization of the NFP as a robust algorithm is, in itself, a highly challenging task. Those considering embarking on research into irregular shaped packing may be discouraged by the significant investment of time required in first developing an NFP generator. Hence, it is essential that robust and easily realizable algorithms are available in order to facilitate new interest into this important problem.

The primary purpose of this paper is to introduce a new procedure for calculating the NFP. The method is developed from the theory of Minkowski sums [7] and builds on the principles proposed by Ghosh [8,9] and by Bennell et al. [10]. Further, the paper includes an algorithmic procedure for eliciting the true boundary of the NFP, including holes, slits and exact fits. It should be noted that the polygons considered in this paper are two-dimensional and restricted to translational motion i.e. rotations are not considered. The next section outlines the most commonly cited approaches for calculating the NFP and points out their positive features and disadvantages. Section 3 reviews in more detail the Minkowski sum approach. This is followed by a description of our new procedure based on Minkowski sums. Section 5, develops our approach for removing redundant internal points and therefore identifying the true boundary. In both cases the full algorithmic steps are provided. Finally, we develop some theoretical and empirical analysis of the approach to demonstrate its robustness with respect to being capable of handling all combinations of simple polygons.

2. Documented approaches for generating the nofit polygon

The NFP is a combination of the properties of two component polygons that, as a result, represents all the relative positions of the two polygons in which they either touch or overlap. It is well documented that the NFP can reduce the complexity of detecting overlap between two pieces from $O(nm + n + m)$, where n and m are the number of edges in each polygon, obtained from direct trigonometry, to a simple point inclusion test of $O(k)$, where k is the number of edges in the NFP. Full explanations of the concept can be found in Mehadevan [2], Ghosh [9], Bennell [11], Bennell et al. [10], and O'Rourke [12], where the most intuitive description is found in Cunningham-Green [13], who describes the motion of one polygon sliding around the boundary of the other; often referred to as the orbiting method. Fig. 1a and b illustrates the motion of polygon B, the orbiting polygon, sliding around A, the fixed polygon, tracing the locus of a reference point on B. He also notes that when both polygons are convex, the NFP is convex and an exact replication of the edges of both polygons, with opposite orientation, sorted into their slope order. Fig. 1c shows the edges of both polygons, where A has counterclockwise orientation and B has clockwise orientation, sorted into slope order; these can be directly mapped onto the NFP in Fig. 1b. Note that this role and orientation of polygons A and B will be adopted for the remainder of the paper. Cunninghame-Green's [13] observations underpin two of the most common approaches to generating the NFP; the orbiting method that simulates the sliding motion, and Minkowski sums that sort the edges according to the slope order and edge precedence, i.e the sequential order of edges around the polygons. A further approach commonly employed is that of decomposition. A brief description of each is provided here.

2.1. Minkowski sum

Clearly, when both component polygons are convex the NFP is very simple to calculate by sorting the edges into slope order. Further, when one of the polygons is convex and the other is an arbitrary simple polygon, the NFP can still