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Strong activity rules for iterative combinatorial auctions

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ABSTRACT

Activity rules have emerged in recent years as an important aspect of practical auction design. The role of an activity rule in an iterative auction is to suppress strategic behavior by bidders and promote simple, continual, meaningful bidding and thus, price discovery. These rules find application in the design of iterative combinatorial auctions for real world scenarios, for example in spectrum auctions, in airline landing slot auctions, and in procurement auctions. We introduce the notion of *strong activity rules*, which allow simple, consistent bidding strategies while precluding all behaviors that cannot be rationalized in this way. We design such a rule for auctions with budget-constrained bidders, i.e., bidders with valuations for resources that are greater than their ability to pay. Such bidders are of practical importance in many market environments, and hindered from bidding in a simple and consistent way by the commonly used *revealed-preference activity rule*, which is too strong in such an environment. We consider issues of complexity, and provide two useful forms of information feedback to guide bidders in meeting strong activity rules. As a special case, we derive a strong activity rule for non-budget-constrained bidders. The ultimate choice of activity rule must depend, in part, on beliefs about the types of bidders likely to participate in an auction event because one cannot have a rule that is simultaneously strong for both budget-constrained bidders and quasi-linear bidders.

1. Introduction

Combinatorial auctions provide a means of auctioning several related items, allowing bidders to place bids on packages of items rather than individual items. They are used in scenarios such as truckload transportation, bus routes, industrial procurement, and allocation of radio spectrum, and have been proposed for the allocation of airport landing slots [1]. Among combinatorial auctions, iterative and especially ascending-price auctions are more widely used than their sealed-bid counterparts, due to the feedback and price discovery that they allow [2].

In high-stakes scenarios, such as auctions for the allocation of wireless spectrum [3] or airport landing slots [4], strategic behavior on the part of bidders can lead to large inefficiency. For example, bidders could underbid in the initial phase of the auction with a view to sniping at the end of the auction, which leads to poor price discovery and inefficient outcomes [5,6]. This necessitates the use of *activity rules* to constraint the strategy space as much as possible while still allowing for feedback and price discovery.

The importance of activity rules in suppressing insincere bidding and eliminating them in iterative auctions has been emphasized in practical auction design [5–7]. Activity rules help in increasing the pace of an auction and increasing the information available to the bidders during an auction. When coupled with careful design of pricing rules, activity rules also help in achieving the efficient outcome with good revenue properties [8–12].

The importance of activity rules has emerged, in part, by the recognition that well-designed iterative auctions should promote simple "demand-revealing processes," whereby bidders simply demand the items that maximize their utility at the current prices. The idea is to promote simple and consistent bidding, or *straightforward bidding*, in which there exists *a posteriori* some (possibly untruthful) utility function that explains the response of bidders in every round.

We introduce the notion of *strong activity rules*, which admit straightforward bidding strategies while precluding all strategies that are not consistent with some straightforward strategy. Strong activity rules do not, in any way, preclude the price discovery and demand discovery benefits associated with iterative auctions. Adopting a strong activity rule does not imply that bidders must either know their utility for different items at the start of the auction, or even bid with the same utility in mind in every round. Rather, a strong rule requires that the bids that they do submit are ultimately consistent with at least one utility function; the set of such utility functions that "rationalize" bidding emerge over time.[1]

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[1] For the particular case of iterative Vickrey auctions, a strong activity rule ensures that truthful, straightforward bidding is an *ex post* Nash equilibrium [10–12].

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One popular activity rule requires that the total quantity demanded by a bidder be monotonically non-increasing as prices increase. However, this is inappropriate for combinatorial auctions because it can preclude straightforward bidding, while on the other hand allow for strategic behavior in which a bidder can bid for a large quantity of low value items and then switch to the items really demanded towards the close of the auction. The *revealed preference activity rule* (*RPAR*) has been proposed as an alternative [5] and advocated for many practical scenarios, including for use in the upcoming UK spectrum auction [13] and for landing slot auctions at New York airports [14].

But many current day markets such as the cellular and airline industries involve budget-constrained bidders [3,15].[2] Budget-constrained bidders have valuations for resources that are greater than their ability to pay, for instance due to liquidity or credit problems.[3] Che and Gale [19] also note that budget constraints can result from a problem of moral hazard; many organizations delegate acquisition decisions to purchasing units, while imposing budgets to constrain their spending.[4] For budget-constrained bidders, RPAR can actually have the opposite effect to that desired, because bidders that bid straightforwardly may fail to satisfy the rule and must instead behave strategically *because* of the rule. In fact, we show that RPAR is also problematic because it fails to guarantee straightforward bidding for bidders without budget constraints.

Both these drawbacks of RPAR illuminate why the design of activity rules needs to be revisited. From the definition of strong activity rules, we are able to develop activity rules for a broadly applicable family of ascending price auctions, allowing for a variety of different prices including non-linear (i.e., the price of a bundle need not be the sum total price of the constituent items) and non-anonymous (or personalized) prices. In the auctions that we consider, the auctioneer reports prices to bidders in each round and bidders respond with a demand set that defines a package of items. Our activity rules also extend to auctions in which the bidder reports multiple packages of items, across which she is indifferent, in each round, and in which the prices are not necessarily ascending. Given this, our results can find application to many auctions. Possible applications include to the *combinatorial clock auction* [20], the *clinching auction* [10,21], RAD [22], *i*-Bundle and ascending-proxy [8,9], the *clock-proxy* auction [5], *i*-BEA [11], *AkBA* [23], and *dVSV* [12].

We explain how to provide feedback to guide a bidder in meeting our strong activity rules, both in terms of the commitments that a bidder is (implicitly) making about her budget constraint through her bids, and also to guide a bidder in modifying bids in order to pass the rule. An auction designer might in fact prefer to provide somewhat relaxed rules; e.g., for reasons of the complexity of the rules themselves, to allow for some mistakes for bidders, and to allow for some value interdependency and learning by bidders. For this, we advocate as a design principle that one should start with a strong rule and then relax this rule as necessary. Relaxing away from our strong activity rule will certainly allow for straightforward bidding behavior but will, in addition, permit some other behaviors.

In practice, we observe that one cannot have a rule that is both strong for budget-constrained and non-budget-constrained bidders, and the choice of rule must depend on the bid-taker's beliefs about the utility functions of participants in the auction.

In experimental simulations, we compare RPAR against our strong activity rule for budget-constrained bidders in the *clock-proxy auction* [5]. This auction is advocated for practical settings such as the FCC wireless spectrum auctions, and consists of an ascending-price combinatorial clock auction phase that terminates with a "last-and-final" round in which bidders submit additional bids before the auction transitions to a sealed-bid (proxy) auction phase. Given our focus on issues related to the activity rule, we assume for the purpose of our simulations that bidders try to bid straightforwardly, and adopt optimization techniques to modify these bids as necessary when this behavior is blocked by the RPAR rule. This is what we refer to as *maximally straightforward* bidding.

Details of our results are provided in Section 5. In summary, the strong activity rule outperforms RPAR with respect to efficiency and revenue by 3.8% and 9.4%, respectively (on average across the different distributions), at low budgets, with benefits falling off as budgets are increased. For certain distributions, we observe efficiency and revenue improvements as high as 13.2% and 20.3%, respectively, while for other distributions the improvements were not statistically significant, even at low budgets.

Organization of the paper: The rest of the paper is organized as follows. In Section 2, we describe the notation and give some preliminaries, and define the notion of a strong activity rule. In Section 3, we discuss existing activity rules and RPAR in particular, describe some of their features and illustrate some key properties that they fail to achieve. We develop the strong activity rules in Section 4 and compare them with other rules, providing also a discussion about extensions and modifications. In Section 5, we discuss our experimental simulations and finally conclude in Section 6.

1.1. Related work

Auctions with budget constraints have been discussed in many works [18,19,24–27]. However, none of this literature discusses activity rules, and every paper is restricted to domains with at most two items for sale. The focus is instead on equilibrium behavior. In the context of combinatorial auctions, impossibility results exist for truthful, Pareto optimal combinatorial auctions in the presence of budget-constrained bidders [28,29]. On the other hand, Pareto optimal and revenue optimal, sealed-bid auctions have been designed for special cases [29,30]. Aggarwal et al. [31] develop a stable, incentive-compatible auction that admits budget-constrained bidders for a generalization of the assignment problem. Ausubel and Milgrom [32] also discuss a generalization of ascending-proxy auctions to allow for budget-constrained bidders. Both of these latter papers are in the context of sealed bid auctions and do not consider the role of activity rules.

Theoretical models for firms with budget constraints allow for both hard and soft (i.e., flexible) budget constraints [33]. Hard budgets are those that cannot be exceeded while soft budget constraints are those that can be exceeded under certain circumstances. Following Kornai et al. [33], a firm can be modeled as hard budget-constrained if it does not receive outside support to cover its deficit and is obliged to reduce or cease its activity if a deficit persists. This is in contrast with soft budgets, where there are supporting agencies that can cover all or part of the deficit; for example, this can be the case for some state enterprises. As the

[2] Airlines typically carry large amounts of debt and are especially vulnerable to fuel spikes, recession or economic shocks [15]. In recent years, all the US airlines together lost over US $35 Billion during the period 2001–2005. Four of the six major US airlines declared Chapter 11 bankruptcy and have only recently emerged [16]. In wireless spectrum auctions, it is realistic to assume that *all* firms participating in these auctions face budget constraints [17,3]. Bidders must raise funds before the auction starts, a time-consuming and costly process that arguably leads to budget constraints.

[3] Pitchik [18] explains that capital market imperfections limit buyers' abilities to borrow against future income when investments are large.

[4] This is observed even for low-valued goods, such as in the domain of sponsored search, in which advertisers can place a limit on the amount they are willing to spend on Internet advertisements over a day.