Science and Justice 56 (2016) 383–387

Contents lists available at ScienceDirect

Science and Justice

journal homepage: www.elsevier.com/locate/scijus

Position paper

An argument against presenting interval quantifications as a surrogate for the value of evidence☆

Danica M. Ommen *, Christopher P. Saunders, Cedric Neumann

South Dakota State University, Department of Mathematics and Statistics, Brookings, SD, United States

ARTICLE INFO

Keywords:
Bayes Factor
Likelihood ratio
Interval
Evidence interpretation

ABSTRACT

In the various forensic science disciplines, recent analytical developments paired with modern statistical computational tools have led to the proliferation of adhoc techniques for quantifying the probative value of forensic evidence. Many legal and scientific scholars agree that the value of evidence should be reported as a likelihood ratio or a Bayes Factor. Quantifying the probative value of forensic evidence is subjected to many sources of variability and uncertainty. There is currently a debate on how to characterize the reliability of the value of evidence. Some authors have proposed associating a confidence/credible interval with the value of evidence assigned to a collection of forensic evidence. In this paper, we will discuss the reasons for our opinion that interval quantifications for the value of evidence should not be used directly in the Bayesian decision-making process to determine the support of the evidence for one of the two competing hypotheses.

1. Introduction

In order to provide context for the presentation of our views on the reasonableness of using intervals for quantifying the value of evidence, we will discuss the structure of the forensic identification of source problem as we typically encounter it. This includes defining the following elements: the structure of the evidence (Section 2.1), the forensic hypotheses under consideration (Section 2.2), the data generating process for the evidence (Section 2.3), and the various methods of quantifying the value of evidence (Sections 2.4, 3, and 4).

In the introduction paper by Morrison [1], there is a list of different types of uncertainty and variability that may be considered when quantifying the value of evidence. For the purpose of the discussion, we will group together the "intrinsic variability at the source", "variability in sampling the relevant population", and "variability in sampling the known source" into *sampling variability of the sources* and discuss the various issues with interval methods used to capture these types of variability in Section 5.2. We will also consider interval methods used to capture the "variability in the statistical modeling technique employed" which we interpreted to mean the *variability due to numerical techniques* in Section 5.1. Finally, we consider the "variability due to modeling assumptions" and the use of sensitivity analysis (as opposed to intervals) to examine this type of variability in Section 5.3. We recognize that the data generating processes described in this paper can be extended to include "variability in the transfer process" or "variability in the measurement technique employed," although we will not explicitly include these in our discussion.

A major concern that we have with using intervals as surrogates for the value of evidence is whether or not it is possible to obtain an admissible decision rule for deciding between the prosecution and defense hypotheses. This is fundamentally a frequentist concern, and is outside the scope of this discussion under the Bayesian paradigm.

2. Forensic identification of source

2.1. The evidence

In the forensic identification of source problem, the evidence can generally be decomposed into three different subsets of observations. The first subset of the evidence is associated with the population of alternative sources (sometimes called the background database/population) and will be denoted e_a. The second subset of the evidence is associated with the trace material recovered from the crime scene, and typically its source is unknown. This subset will be denoted e_u. The third subset of the evidence is associated

☆ This paper is part of the Virtual Special Issue entitled: Measuring and Reporting the Precision of Forensic Likelihood Ratios, [http://www.sciencedirect.com/science/journal/13550306/vsi], Guest Edited by G. S. Morrison.

* Corresponding author.
E-mail address: danica.ommen@sdstate.edu (D.M. Ommen).

http://dx.doi.org/10.1016/j.scijus.2016.07.001
1355-0306/

with control material taken from a known specific source, typically the suspect, and it will be denoted e_s. The collection of all the evidence will be denoted by $e = \{e_a, e_u, e_s\}$.

2.2. The hypotheses

A common question regarding the forensic identification of source problem that is asked is:

"Does the trace material originate from the known specific source or from some other source in the alternative source population?"

Directly related to this question is a set of forensic hypotheses. The first hypothesis is often referred to as the prosecution hypothesis (denoted H_p), and the second is referred to as the defense hypothesis (denoted H_d). For the forensic hypotheses related to the question posed above, H_p states that the trace and the control samples were both generated by the specific source, and H_d states that the trace was not generated by the specific source, but by some other source in the relevant alternative source population.

2.3. Data generating process

The data generating process for the subsets of the evidence described above must be specified in order to ultimately compute the value of evidence. These assumptions will be described in a traditional sampling framework [2], although they will also imply a corresponding set of exchangeability assumptions made under a fully Bayesian framework. The data generating process for the forensic identification of source problems suggests that each subset of the evidence is generated as a random sample from a population which can be modeled with a parametric family of distributions characterized by a (unknown) set of parameters. We will denote the full set of parameters indexing the parametric family of distributions for the entire set of evidence e as θ, and when the values of θ are known, this set of values will be denoted θ_0. Once the parametric family of distributions for each subset of the evidence has been chosen, then the likelihood functions, denoted by f, for the subsets of the evidence have been determined.

For the hypotheses described above, the subsets of the evidence $e = \{e_a, e_u, e_s\}$ are three independent samples drawn in the following way:

1. e_a is constructed by first taking a simple random sample of sources from a given relevant population of alternative sources; then from each sampled source we have a simple random sample. Let θ_a denote the parameters necessary to describe this sampling-induced distribution.
2. e_u is a simple random sample from a single source.
3. e_s is a simple random sample from a given specific source. Let θ_s denote the parameters necessary to describe this sampling-induced distribution.

Under H_p the source of e_u is the specific source from 3. and the sampling distribution of e_u is characterized by the parameters θ_s. This statement in combination with the data generating process described in points 1.–3. above will be denoted M_p. Under H_d, e_u arises from a randomly selected source in the alternative source population and the sampling distribution of e_u is characterized by the parameters θ_a. This statement in combination with the data generating process described in 1. - 3. above will be denoted M_d.

The application of Bayesian methods to these problems requires that the prior probability densities for the parameters be specified; one summarizing our belief about the values of the parameters for the specific source data generating process, $\pi(\theta_s)$, and another summarizing our prior belief about the values of the parameters for the alternative source population data generating process, $\pi(\theta_a)$.

2.4. Quantifying the value of evidence

Under the Bayesian framework, the forensic statistician is tasked with providing the value of evidence that is used to update a prior belief structure concerning the two competing hypotheses. Traditionally, the value of evidence is used to convert the prior odds to posterior odds as follows:

$$\underbrace{\frac{P(H_p|e,I)}{P(H_d|e,I)}}_{\text{Posterior Odds}} = \underbrace{\frac{P(e|H_p,I)}{P(e|H_d,I)}}_{\text{Value of Evidence}} \times \underbrace{\frac{P(H_p|I)}{P(H_d|I)}}_{\text{Prior Odds}}, \quad (1)$$

where e is the realization of the evidence, I is the relevant background information common to both hypotheses, H_p and H_d, and P is some probability measure. The value of evidence is typically known as a "Bayes Factor" in the statistics community and a "likelihood ratio" in the forensic science community [3]. However, we consider the Bayes Factor and likelihood ratio to be two distinct quantities. In particular, when the values of the parameters for the data generating processes are known with certainty, the value of evidence takes the form of the likelihood ratio, which will be described in detail in Section 3. When there is uncertainty concerning the values of the parameters, other strategies need to be used to quantify the value of evidence. One of the commonly used strategies takes the form of the Bayes Factor, described in detail in Section 4.

3. The likelihood ratio

We define the likelihood ratio function as

$$\lambda_{e_u}(\theta) = \frac{f(e_u|\theta, M_p)}{f(e_u|\theta, M_d)}, \quad (2)$$

which is a function of the unknown parameters indexing the data generating process. When there is no uncertainty about the parameters of the data generating process, the likelihood ratio is

$$\lambda_{e_u}(\theta_0) = \frac{f(e_u|\theta_0, M_p)}{f(e_u|\theta_0, M_d)}, \quad (3)$$

and represents a single value of the likelihood ratio function for the specified values of θ. Indeed, the likelihood structure, denoted f, and the values of the parameters, denoted θ_0, for these models need to be known with complete certainty to compute the likelihood ratio [4,5]. The exact form of the likelihood ratio for the identification of source problem described in Section 2 is presented below.

3.1. Specific source likelihood ratio

Under H_p, the unknown source evidence is characterized by the same model and parameters characterizing e_s. Therefore, the numerator of the likelihood ratio, denoted $\lambda_{e_u}(\theta_0)$ below, will be the likelihood of observing e_u given that the value of the parameter θ_s is θ_{s_0}. Similarly, under H_d, e_u is characterized by the same model and parameters as that of e_a. Therefore, the denominator will be the likelihood of observing e_u given that the value of the parameter θ_a is θ_{a_0}. The likelihood ratio

$$\lambda_{e_u}(\theta_0) = \frac{f(e_u|\theta_{s_0})}{f(e_u|\theta_{a_0})}, \quad (4)$$

when it can be determined, is fixed and everyone with comparable beliefs would agree that it is the value of evidence that should be used in Eq. (1). It should be noted that this likelihood ratio is rarely attainable in practice, and some choose to estimate it using various techniques. Typical adhoc methods are based on using maximum likelihood estimates, restricted maximum likelihood estimates, or combining