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Hierarchical maximal-coverage location–allocation: Case of generalized search-and-rescue

Yupo Chan[a,*], Jean M. Mahan[b], James W. Chrissis[c], David A. Drake[d], Dong Wang[e]

[a]Department of Systems Engineering, University of Arkansas, 2801 South University, Little Rock, AR 72204-1099, USA
[b]Transportation Command, USA
[c]Air Force Institute of Technology, USA
[d]Science Applications International Corporation, USA
[e]Arizona State University, Tempe, USA

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Abstract

We offer a variant of the maximal covering location problem to locate up to p signal-receiving stations. The "demands," called geolocations, to be covered by these stations are distress signals and/or transmissions from any targets. The problem is complicated by several factors. First, to find a signal location, the signal must be received by at least three stations—two lines of bearing for triangulation and a third for accuracy. Second, signal frequencies vary by source and the included stations do not necessarily receive all frequencies. One must decide which listening frequencies are allocated to which stations. Finally, the range or coverage area of a station varies stochastically because of meteorological conditions. This problem is modeled using a multiobjective (or multicriteria) linear integer program (MOLIP), which is an approximation of a highly nonlinear integer program. As a solution algorithm, the MOLIP is converted to a two-stage network-flow formulation that reduces the number of explicitly enumerated integer variables. Non-inferior solutions of the MOLIP are evaluated by a value function, which identifies solutions that are similar to the more accurate nonlinear model. In all case studies, the "best" non-inferior solutions were about one to four standard deviations better than the sample mean of thousands of randomly located receivers with heuristic frequency assignments. We also show that a two-stage network-flow algorithm is a practical solution to an intractable nonlinear integer model. Most importantly, the procedure has been implemented in the field.

Keywords: Maximal-coverage facility-location; Location–allocation; Primary vs. backup coverage; Assignment; Mutiobjective optimization; Network flow; Enhanced 911

1. Introduction

In locating a vehicle, ship or airplane in distress, listening posts are used to provide lines-of-bearing (LOB). Through triangulation, they provide the general vicinity from where the distress signal is transmitted. Distress signals are transmitted at an agreed upon frequency so that search-and-rescue (SAR) operations can be launched quickly [1]. Consequently, a receiver has to be tuned to the right frequency to detect a distress signal. In this paper, we are concerned with the generalized SAR (GSAR) problem, where the transmission frequency is not necessarily preset. This means

* Corresponding author. Tel.: +1 501 569 8926; fax: +1 501 569 8698.
E-mail address: ychan@alum.MIT.edu (Y. Chan).

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that the receiving stations are responsible not only for locating ships and planes, but also any targets (including hostile targets). As a result, the frequency can vary within the transmission frequency spectrum. In this paper, the transmitter areas are arbitrarily selected points along established merchant ship routes where the SAR operations are likely to occur.

For the GSAR problem, we must locate receiving stations. Then we allocate high-frequency direction-finder (HFDF) equipment at receiving stations, and assign listening frequencies to these HFDFs. An HFDF yields a compass bearing from a received distress signal. The objective of the location/allocation problem is to pinpoint or geolocate as many targets, both in distress and otherwise, as possible. Besides commercial applications, the GSAR problem has obvious application to military intelligence [2–4]. It can also be employed to place such civilian navigation systems as air-traffic-control centers, cell towers and enhanced-911 receivers [5–8].

There are two types of detection systems at a receiving station: the receiving equipment and the HFDFs. While receiving equipment scans all frequencies to detect a target, an HFDF monitors just a preset frequency band to confirm the target previously identified. An HFDF is more sensitive than a receiving equipment and can receive weaker signals. Receiving equipment is usually used to *acquire* a signal while an HFDF used to *confirm* the signal. HFDFs are assigned in bundles to a receiving station to cover the desired frequencies, with each HFDF responsible for a particular frequency band. With one receiving equipment and eight HFDFs located at each receiving station, the problem of assigning listening frequencies to HFDFs is a combinatorially explosive task. Given 30 possible listening frequencies and eight HFDFs at a single station, there are more than five million possible *frequency* assignments for a single station. For the case where 10 receiving stations are to be chosen from 25 candidate sites, there are more than three million possible *locational* arrangements. The combination of possible location–allocation *and* frequency assignments jumps to well over 1056 million! Clever combinatorial optimization techniques are required to solve this problem, since total enumeration is both impractical and impossible, even with emerging computers.

Now consider the transmission probabilities of distress signals changing from one time period to another due to varying meteorological conditions. To the extent that transmission frequencies can change twelve times within a day, we need an efficient solution algorithm to update these frequency assignments on a real-time basis. Frequency assignments among HFDFs have to be flexible and changed from time period to time period. However, HFDF locations should cater to all time periods since in the short run it is impossible to re-locate HFDFs from one receiving station to another. The GSAR problem tries to identify the optimal location of HFDFs for all transmission time periods, but with listening frequencies being re-assigned from period to period.

2. The GSAR problem as a location problem

For the GSAR problem, we consider the locational pairing of receiving stations j, where $j = 1, \ldots, J$, to distress signals i, where $i = 1, \ldots, I$. Since one does not know a priori when and where a distress will occur, we cover a study area with a comprehensive set of discrete locations where a distress signal can originate. In facility-location modeling, such pairing of a facility j to a demand i can be formulated as a maximal-coverage location problem that locates p facilities ($p \leqslant J$) to maximize the coverage of I demands. Since there are a limited number of available HFDFs, the location problem is *capacitated*. Here the capacitated maximal-coverage (CMC) facility-location problem is a generalization of the *multiproduct* extension of the classical coverage models. To model the GSAR problem, *four* extensions of the CMC problem are incorporated.

First, more than one receiving-station location j is assigned to a single distress-signal location i, allowing triangulation coverage for each i. Second, a distress-signal location, i, transmitting on frequency $k (k = 1, \ldots, K)$ must be paired to a receiving-station location, j, with an HFDF at the station assigned to frequency k. This extension increases the dimensionality of the problem, since the running index i–j now becomes i–j–k. Third, more than one HFDF can be located at each site j, covering multiple frequencies. Specifically, HFDFs are assigned to each receiving-station location j in integer bundles of size n_j, where n_j is exogenously determined. Fourth, we define P as the probability that an HFDF on frequency k at receiving-station j *cannot* respond to a distress signal i on frequency k. Since this probability is a reflection of the inherent limitation of the equipment, P is assumed to be identical and independent across all facilities and demands. This is called a probabilistic maximal-coverage problem [9,10,30,32].

Snyder and Daskin [11] propose to choose facility locations that are both inexpensive under traditional objective functions and also *reliable*. Hogan and Revelle [12] suggest a facility-location model to handle stochastic demands. Backup coverage is proposed in areas of high demand to maintain a uniform level of service. Backup coverage can be