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Review

Representing Something Out of Nothing: The Dawning of Zero

Andreas Nieder[1,*]

Zero stands for emptiness, for nothing, and yet it is considered to be one of the greatest achievements of humankind. This review first recapitulates the discovery of the number zero in human history, then follows its progression in human development, traces its evolution in the animal kingdom, and finally elucidates how the brain transforms 'nothing' into an abstract zero category. It is argued that the emergence of zero passes through four corresponding representations in all of these interrelated realms: first, sensory 'nothing'; then categorical 'something'; then quantitative empty sets; and finally the number zero. The concept of zero shows how the brain, originally evolved to represent stimuli ('something'), detaches from empirical properties to achieve ultimate abstract thinking.

> The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought. (Alfred North Whitehead)
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> In the history of culture the discovery of zero will always stand out as one of the greatest single achievements of the human race. (Tobias Dantzig)

Special Status of Zero among the Natural Numbers

Any number is an abstraction, a recognition that collections may have something in common even if the elements of the collections are very different [1,2]. The number 2 is the common property of all sets containing a pair, the number 3 of all sets that contain a triple, and so on [3]. However, although they are abstract and demanding, positive integers correspond to real 'things' that can be enumerated [4]. Therefore, we first learn to count small numbers of items and later use this counting procedure to comprehend infinite positive numbers [1].

Zero, however, does not fit into this routine (Box 1). While the counting procedure is based on the assumption that there is something to be counted, a set with no elements cannot be assessed via counting. Understanding that zero is still a collection (even if empty) and a numerical concept requires abstract thinking that is detached from empirical experience [5]. The problem is that 'nothing' needs to become 'something'. The absence of elements needs to become a mental category – a mathematical object [1,6].

As a reflection of this mental challenge, it took a long stretch of human history for zero to be recognized and appreciated [7,8]. This cultural hesitation is mirrored in a protracted ontogenetic understanding of numerosity zero in children [9]. Unsurprisingly, only advanced nonhuman animals with which we share a nonverbal quantification system exhibit rudiments of a grasp of

Trends

Recent studies in human history, developmental psychology, animal cognition, and neurophysiology provide evidence that the emergence of zero passes through four stages.

In the first and most primitive stage, the absence of a stimulus ('nothing') corresponds to a (mental/neural) resting state lacking a specific signature.

In the second stage, stimulus absence is grasped as a meaningful behavioral category but its representation is still devoid of quantitative relevance.

In the third stage, nothing acquires a quantitative meaning and is represented as an empty set at the low end of a numerical continuum or number line.

Finally, the empty-set representation is extended to become the number zero.

These different stages of zero-like concepts reflect progressing levels of mental abstraction and pave the way for a full-blown number theory.

[1]Animal Physiology Unit, Institute of Neurobiology, University of Tübingen, Auf der Morgenstelle 28, 72076 Tübingen, Germany

*Correspondence: andreas.nieder@uni-tuebingen.de (A. Nieder).

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Box 1. Zero and the Empty Set

Definition and Features of Zero [3,73]

- Zero is a number to describe 'no quantity' or 'null quantity'.
- Zero is the only natural number (by most definitions) that is not positive.
- Zero is neither positive nor negative.
- Zero is the integer immediately preceding 1.
- Zero is an even number because it is divisible by 2.
- Zero is the only number that can be divided by every other number.
- Zero is the only number that can divide no other number.
- Zero is a prerequisite to understanding negative numbers.
- Beyond denoting null quantity, zero has a second and discrete function as a placeholder sign (or digit) in other numbers (e.g., 30, 103).

Empty Set in Set Theory

- Set theory is the mathematical theory of well-determined collections, called *sets*, of objects that are called *members*, or *elements*, of the set. The size of a set (its number of elements) is called its *cardinality*.
- A set that contains no elements is called an empty set or a null set and is denoted by $\varnothing$ or {} (null-set axiom of Zermelo–Fraenkel set theory); for instance, if set $A = \{2, 3, 4\}$ and $B = \{5, 6, 7\}$, then $A \cap B = \{\}$.
- The empty set is not the number 0.
- The empty set is not nothing, because a set containing no element still is a set [74].

Glossary

Numerical-distance effect: psychophysical phenomenon of magnitude discriminations; the greater the magnitude difference between two numerosities, the more easily they can be discriminated.

Place-value systems: numerals adopt different numerical values according to their position in a record. Each position is related to the next by a constant multiplier; for example, units, tens, hundreds, and so on.

Positional notation system: see place-value systems.

Prefrontal cortex (PFC): associative cortical region in the anterior frontal lobe of mammals central to cognitive control (executive) functions and high-level cognition.

Single-cell recordings: measurement of the electrical action potentials of single neurons as physical carriers of information in the brain.

Ventral intraparietal area (VIP): associative cortical area in the fundus of the intraparietal sulcus of the parietal lobe of primates.

zero numerosity [10]. For a brain that has evolved to process sensory stimuli (something), conceiving of empty sets (nothing) as a meaningful category requires high-level abstraction. It requires the ability to represent a concept beyond what is perceived.

The Four Stages of Zero-Like Concepts

Until recently, the biological origins of the understanding of zero were unknown. Recent studies from human history, developmental psychology, animal cognition, and neurophysiology provide evidence that the emergence of zero passes through four stages. These four stages are used to structure this review (Figure 1, Key Figure). In the first and most primitive stage, the absence of a stimulus (nothing) corresponds to a (mental/neural) resting state lacking a specific signature. In the second stage, stimulus absence is grasped as a meaningful behavioral category but its representation is still devoid of quantitative relevance. In the third stage, nothing acquires a quantitative meaning and is represented as an empty set at the low end of a numerical continuum or number line. Finally, the empty set representation is extended to become the number zero, thus becoming part of a combinatorial number symbols system used for calculation and mathematics. As outlined in the following, these different stages of zero-like concepts reflect progressing levels of mental abstraction.

Zero in Human History

The number zero is a surprisingly recent development in human history [7]. Zero was first used as a simple sign to indicate an empty place for integers in a so-called **positional notation system** (see Glossary) (**place-value system**). In a positional notation system, a numeral has different numerical values according to its position in a record: units, tens, hundreds, and so on. For instance, the 3 in the number 302 stands for three hundreds, whereas in the number 203 it denotes three. Without a sign for an empty space, any place-value number record is highly ambiguous. This can be seen, for instance, when in China around 500 BC [11] rod numerals were placed on the columns of a counting board to represent digits in a base-ten decimal system and perform calculations (Figure 2A). Zeroes were represented by an empty space [8]. The problem with this notation is that an entry such as ||| | may represent any one of several numbers: 31, 301, or 310 among others.

Zero as a sign for an empty column in positional notation appears to have first been used around 400 BC in ancient Mesopotamia by the Babylonians, who used two slanted wedges as a place holder (Figure 2B) [7,11–13]. Slightly later, the Greeks used a circle as a placeholder, probably