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Product selection, machine time allocation, and scheduling decisions for manufacturing perishable products subject to a deadline

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Abstract

We are concerned about the following problem: a manufacturer owns a certain amount of perishable raw material which can be produced into different types of products. He must, however, finish the manufacturing process before a deadline (which represents, e.g., a fixed flight schedule). Due to the deadline constraint and the raw material limit, it is imperative for the manufacturer to determine three decisions: (i) the product types to be produced; (ii) the machine time to be allocated for each product type; and (iii) the sequence to process the products selected. We develop, in this paper, a model to formulate this problem. We show that (i) and (iii) can be determined by analytical rules, and (ii) can be computed by an efficient algorithm. The optimal policy with the three decisions for the problem is therefore completely constructed. We also show the relationships of our model to stochastic scheduling and stochastic knapsack, and discuss the contributions of our work to the two areas.

Keywords: Production planning; Product selection; Stochastic scheduling; Deadline; Machine time allocation; Perishable product

1. Introduction

Our study is motivated by the following problem: at the beginning of each day, a seafood manufacturer receives from the local fishermen a certain amount of raw fish, which can be processed into different types of seafood products. To maintain freshness, the finished seafood products are to be transported to a distributor by air on the same day. The flight is operated, however, on a fixed schedule, and any raw material that is not used before this delivery time will have little salvage value due to its perishable nature and the freshness requirement of the products. Given the constraints of the raw material and the delivery deadline, how to make the best production decisions, including the types and volumes of products to be manufactured and the schedule to process the products, so as to maximize the total profit or minimize the total loss? This is a critical problem facing the manufacturer.

We develop, in this paper, a model to formulate the problem above, which focuses on joint selection/scheduling decisions, and takes into account product-specific profit margins, stochastic processing time requirements, and resource

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(raw material) and timing (deadline) constraints. Specifically, the model we develop and investigate can be described as follows: a set of jobs are to be selected and processed into products on a single machine. Each job consumes a certain amount of raw material and requires a random processing time. The objective is to minimize the total expected cost, by determining an optimal solution comprising three decisions: (1) the subset of jobs to be selected for processing; (2) the amount of machine time to be spent to process each selected job; and (3) the sequence to process the selected jobs on the machine. The optimal solution must satisfy two constraints: (i) The limit on the raw material, which may lead to the scenario that jobs sequenced earlier may use up all raw material and therefore jobs sequenced later may not be produced; and (ii) The deadline to complete the products, after which all unfinished jobs will become valueless and the raw material left will become useless.

Although this model originates from the seafood production scenario as described above, it is applicable to other situations involving uncertain deadlines or limited resources. An example is the problem where a subset of tasks, each requiring a processing time that is uncertain or random, have to be selected to complete when a deadline is known to approach—this corresponds to our model with a deadline but without the raw material constraint. Another example is the case where the resource is limited, but the deadline is not restrictive. The concern in such a case is how to optimize the division of the resource to a number of tasks so as to maximize the total profit. We will elaborate, in one of the sections to be presented below (See Corollaries 1 and 2 of Section 3.2), how our main model and the associated optimal policies can be applied in these situations.

Our model can be viewed to relate broadly to two categories of problems: stochastic knapsack and stochastic scheduling. The product selection decision of our model is a knapsack problem. After the products to be processed have been selected, the sequencing decision of our model is a stochastic scheduling problem. We now review the relevant literature.

The job (product) selection decision of our model is a stochastic knapsack problem, in the sense that the "delivery time" is the total size of the knapsack, and the "processing time" of a job is its size. The objective is to determine which jobs should be selected so that the total processing time is not greater than the delivery time while the total expected profit is maximized. It is a stochastic knapsack problem, because the value a job can generate if it is selected is random. (Due to the random processing time of the job, it is uncertain how many quantities of a product can be produced in a given time slot and thus the profit that the selected product can generate is not certain.) Stochastic knapsack problems with objects of deterministic sizes and random values have been studied extensively in the literature; See, e.g., Steinberg and Parks [1], Kleywegt and Papastavrou [2], and Lu et al. [3]. Our work is different from the traditional stochastic knapsack studies, in the sense that the objective function of our model becomes highly nonlinear and nonseparable after we take the expectation (see the objective function of Problem P2 in Section 3.2 below). As pointed out by Bretthauer and Shetty [4], such kinds of problems are "much more difficult to solve than the separable problem". An interesting aspect of our work here is that we can, by an appropriate transformation, turn the problem into a separable one and thus obtain the optimal solution analytically.

After the subset of jobs to be processed have been selected, our model falls into the framework of stochastic scheduling. This is because of the randomness involved in the processing times and the job sequence to be determined. Pinedo [5] contains comprehensive review of stochastic scheduling problems with regular objectives (which are increasing functions of job completion times); and Cai and Zhou [6] reviews stochastic scheduling problems with irregular performance criteria. The cost function of our model is regular, in the sense that the later a job completion, the higher is its cost (a later job may face the risk that the raw material left is not sufficient). A main result we obtain in this paper is that the optimal sequence of our problem follows the LEPT order (the order of longest expected processing time first). It is interesting that the LEPT order is optimal for a single-machine stochastic scheduling problem with a regular cost function.

The model developed in this paper is also relevant to two other streams of research in the scheduling literature. One is scheduling models with batching. By batch production, the manufacturer achieves a scale of economies. There are reviews of models which combine scheduling with batching; see, for example, Potts and Van Wassenhove [7], Webster and Baker [8], and Potts and Kovalyov [9]. When the manufacturing types are given, our model is a single machine scheduling model with batching and zero setup cost. However, different from most of the existing literature on scheduling with batching, the total machine time allocated for each batch in our model is a decision variable while the number of jobs in each batch is a random variable depending on the allocated machine time. We also assume that each batch of jobs are subject to a common deadline/due date in our model, a situation that may appear in many realistic problems (see, e.g., review by Baker [10]).