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PERM for solving circle packing problem

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Abstract

In this paper, we develop a new algorithm that incorporates the improved PERM into an already existing simple deterministic heuristic, the principle of maximum cave degree for corner-occupying actions, to solve the problem of packing equal or unequal circles into a larger circle container. We compare the performance of our algorithm on several problem instances taken from the literature with previous algorithms. The computational results show that the proposed approach produces high-quality solutions within reasonable computational times. Although our algorithm is less efficient than Zhang's for several large-scale equal-size instances, it is noteworthy that for several unequal circle instances we found new lower bounds missed in previous papers.

Keywords: Circle packing; PERM; Corner-occupying action; Cave degree; NP-hard

1. Introduction

The packing problem is concerned with how to pack a number of objects, each with given shape and size, into a bounded space without overlap. Many optional features exist on this problem, e.g. the container can be rectangle, circle or polygon and the radii of circles can be equal or different. In this paper, we consider the problem of packing a given set of circles with equal or unequal radii into a larger containing circle. Since this problem has proven to be NP-hard [1,2], the combinatorial complexity makes it intractable as soon as the search space becomes large. Thus, heuristic methods are the only feasible means to provide solutions for this kind of problems.

Iserman [3], Fraser and George [4] and Dowsland [5] developed various heuristics for packing equal-size circles into a rectangle. George et al. [6] studied the problem of packing unequal-size circles into a rectangle using some heuristic rules and different combinations of these rules. Huang and Kang [7] proposed a quasi-physical strategy for packing equal circles into a triangular container. Stoyan and Yaskov [8] presented a mathematical model and a solution method combining the branch-and-bound algorithm with the reduced gradient method to pack unequal-size circles into a strip. Huang and Xu [9] proposed a quasi-physical and quasi-human approach to simulate the movement system for packing unequal and equal circles into a circle container. Zhang and Deng [10] proposed a hybrid algorithm by combining simulated annealing scheme with tabu search strategy to pack unequal and equal circles into a circle container. Mladenović et al. [11] proposed a reformulation decent method to pack equal circles into a unit circle. Hifi and M'Hallah [12] developed a constructive procedure-based heuristic and a genetic algorithm-based heuristic to study the problem of cutting a rectangular plate into as many circular pieces as possible.

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Recently, Huang et al. [13,14] proposed a heuristic, the principle of maximum cave degree for corner-occupying actions (COAs), to select and pack the circles one by one, and they proposed a two level search strategy to improve the basic heuristic algorithm. In addition, Pruned–Enriched-Rosenbluth Method (PERM), also called population control algorithm, is a powerful strategy for pruning and enriching branches when searching the solution space and it has shown to be very efficient for solving protein folding problem [15,16]. In this paper, we present a new method that incorporates the PERM scheme into the strategy of maximum cave degree. The basic idea of our approach is to evaluate the benefit of a partial configuration (where some circles have been packed and others outside) using the principle of maximum cave degree, and use the PERM strategy to prune and enrich branches efficiently.

The paper is organized as follows. A mathematical formulation of the circle packing problem is given in Section 2. In Section 3, the existing heuristic "the principle of maximum cave degree", the PERM strategy and the combination of these two strategies are proposed. Numerical results are presented and compared with other existing methods in Section 4. Section 5 concludes the paper.

2. Circle packing problem

Given the radii of n circles and the radius of the larger circle container R_0, the problem consists of placing these circles into the container without overlap, if possible. We formulate the circle packing problem as follows:

In a 2D Cartesian coordinate system, the coordinate of the center of the large containing circle is fixed at (0, 0) and the coordinate of the center of the ith circle is denoted by (x_i, y_i). r_i denotes the radius of circle i. The problem is to determine if there exist a set of real numbers $(x_1, y_1, \ldots, x_n, y_n)$, such that

$$\sqrt{x_i^2 + y_i^2} \leqslant R_0 - r_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geqslant r_i + r_j, \quad i, j = 1, 2, \ldots, n, \ \ i < j. \tag{2}$$

Constraint (1) denotes that each circle in the container should not extend outside the container. Constraint (2) requires that any pair-wise circles placed in the container cannot overlap each other.

3. The proposed algorithm

Before describing the algorithm, we first give some preliminaries about the efficient heuristic [13,14], the principle of maximum cave degree, for packing circles into a larger circle container. Then we propose a placement heuristic called A0 to quickly pack the circles into the container. Subsequent PERM search strategy is presented and combined with the above-mentioned heuristic to solve the circle packing problem.

3.1. Preliminaries

Our proposed algorithm is based upon "growth" strategy, e.g. all the circles are selected and packed into the container one by one according to some heuristic rules.

Definition 1 (*Pattern*). A pattern is a partial configuration where m ($m \geqslant 2$) circles have been already packed inside the container without overlap and $n - m$ circles remain to be packed.

Definition 2 (*Corner-occupying action (COA)*). Given a pattern, a legal COA is the placement of circle inside the container so that the circle does not overlap with any other packed circle and is tangent with two *objects* in the container (one of the two *objects* may be the container itself or an already packed circle).

For circle i, a COA is denoted by (i, x, y), which means that circle i is to be placed at position (x, y). Note that there may be several COAs for a single circle i, denoted by (i, x_{i1}, y_{i1}), (i, x_{i2}, y_{i2}), …, (i, x_{it}, y_{it}). In Fig. 1, the three dotted circles denote the COAs for circle 6 under the pattern where five circles have been packed.

For a pattern where m circles have been packed, the number of all possible COAs for a single circle is bounded by $O(m^2)$ because there are C_{m+1}^2 pair-wise circles (including the container) and the total number of COAs for $n - m$ unpacked circles are bounded by $O((n - m)m^2)$.