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Original Research Paper

Stereological correction method based on sectional texture analysis for the liberation distribution of binary particle systems

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ABSTRACT

Accurate assessment of liberation state of ore sample is important in mineral processing. In practice, the ore sample is mounted in resin, sectioned, and polished before its liberation state is measured by sectional analysis. This approach typically overestimates the degree of liberation in two-dimensional (2D) measurements. Several models have been proposed to overcome this *stereological bias* and correct this error in well-examined samples. However, their versatility remains poorly understood. Herein, a stereological correction method was developed for the liberation distribution. First, the complexity of particle sectional texture was assessed using the fractal dimension of the image intensity. Next, the magnitude of stereological bias and 2D measureable parameters such as fractal dimension was correlated by an all-encompassing simulation of various texture types. Finally, stereological correction indices were exclusively estimated from 2D measureable parameters obtained through the correlation. This model exhibited high versatility based on the all-encompassing simulation. The model was validated using nine different irregularly shaped binary particle systems and assessed using areal difference and maximum difference between liberation distribution curves in two and three dimensions. This error indices improved by approximately 80% for the former error index and 90% for the latter index.

1. Introduction

In mineral processing, comminution mainly aims at freeing valuable minerals from gangue to enhance the efficiency of the subsequent selection and consequently, the total efficiency of the entire process. The fraction of liberated particles with respect to the total amount of the phase corresponds to the *degree of liberation*, which is an important index in the assessment of the comminution process. In practice, a perfect liberation is not always necessary but particles composed of the phase of interest may need to achieve a certain content (e.g., 95%). Therefore, the degree of liberation and the *liberation distribution*, which represents the distribution of the content of phase of interest are of great concern.

In general, the liberation distribution, also known as *volume grade distribution* and *liberation spectrum*, is obtained by 2D examining resin mounted, sectioned, and polished particle sections. These two-dimensional (2D) observations are conducted using a traditional microscope [1] or scanning electron microscopy/energy-dispersive X-ray spectroscopy (SEM/EDX) based automatic analyzer such as mineral liberation analyzer [2]. This 2D approach inevitably result in *stereological bias* in the liberation distribution, because a liberated particle always appears like a liberated section, whereas a multi-phased particle may appear as liberated section [3].

Some approaches have been proposed to avoid this error and directly obtain three-dimensional (3D) liberation information by X-ray computed tomography [4–8] and serial particle sectioning [9–11]. These approaches have proven successful at a research stage but have not been applied because of inadequate analysis speed, cost, and accuracy of mineral identification. Therefore, *stereological correction*, in which the 3D liberation distribution is estimated from 2D data obtainable by sectional analysis, has attracted extensive interest.

Miller and Lin [11] and King and Schneider [12] proposed a correction method converting 11 or 12 classes of 2D liberation distribution into their 3D counterparts using a kernel function. This method requires a separation test with a high degree of accuracy to determine the kernel function. Given that the kernel function may be influenced by the particle's texture and being unrealistic to determine the kernel for each mineral sample, a systematic study on the influence of the particle texture on the kernel function is required. Gay and Morrison [8] took another approach to

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determine the kernel function. The relationship between the line assessment (1D) and the area assessment (2D) is applied to estimate the kernel function to determine the 3D liberation from the area assessment. This method enables us to obtain all the parameters from sectional analysis without a separation test. However, further study is necessary for the relationship between the kernel functions of 1D to 2D and 2D to 3D.

Barbery [13] established a correction method using particle structure models of the Poisson mosaic and the Boolean and particle shape model based on the line assessment. From the probability calculation using these models, four parameters, viz. grade, uniformity of particle composition, the liberation functions of two phases are determined. The 3D liberation distribution is estimated using the four parameters with the gamma function. This approach enables us to obtain all the parameters from the sectional analysis. According to the validation study using the hematite-quartz ore, the estimated degrees of liberation successfully fit the 3D values [13].

Gay has produced a remarkable achievement in this topic and proposed many correction models [8,14]. His representative model is summarized and validated in Ref. [15]. The section feature of each particle is assessed by 18 parameters: phase area, distance of two pixels, area of three pixels triangle, perimeter, and surface area with the combination of two phases. The particles are allocated to the twelve classes of 3D liberation distribution with satisfying the nine stereological equations. An objective function is used to minimize the error in the equations. It may be the most accepted method currently. Further validation studies are expected for the model's applicability to particle systems with various types of liberation distributions.

The stereological bias has strong association with the internal structure of particles [9,16]. Recently, it was found that the feature of 3D internal structure of particles that controls the stereological bias can be estimated by 2D texture analysis of the particle sections [17]. A correction method based on this knowledge has been established [18]. First, a database of the association between the stereological bias and the 2D sectional information was established by a series of encompassing simulations on various types of binary particle models. Second, the texture of sample particle sections was assessed by a texture analysis method (e.g., a fractal dimension [17]), and the stereological bias was estimated by the database. Third, the 3D degree of liberation was predicted using the estimated stereological bias and the measured 2D degree of liberation. In this method, the stereological bias on the degree of liberation (0% or 100% of mineral of interest) was investigated, and as an extension, that of the liberation distribution (0–100% of mineral of interest) was examined and its correction method was proposed in this study.

In particular, a series of stereological bias analyses were conducted using more than one million particles with 1,470 different textures. Next, correlations were calculated between the stereological bias and 2D measureable parameters for 12 classes of liberation distribution. Finally, a stereological correction model was proposed for the liberation distribution and validated using nine types of irregularly shaped particles, thereby establishing the versatility of the stereological correction method for the liberation distribution and making it applicable to various particles.

2. Methodology

2.1. Binary particle modeling

Binary particles comprising a core phase A and matrix phase B were modeled by a known procedure [19]. Briefly, we observe the following:

(A) 7463 spherical particles with diameters ranging from 1.0 to 2.0 were generated at random positions in a cuboid ($30 \times 30 \times 15$) and packed in freefall by the Discrete Element Method (DEM) [20] (Fig. 1), implemented in the open source software ESyS-Particle [21].

(B) Spherical *phase A elements* with a diameter (d_A) and a volume fraction (F_v^0) (the ratio of total volume of the phase A elements to volume of the cuboid) were created at random positions in the cuboid, independently from the particles. Therefore, these elements could freely overlap with particles.

(C) Particle domain overlapping with phase A elements were grouped in the *phase A domain*, whereas the rest were incorporated in the *phase B domain*.

(D) Steps B) and C) were repeated 250 times with fixed values of d_A and F_v^0 but random phase A element positions to prepare a large number of binary particles.

2.2. Volume and sectional area calculation

Two- and three-dimensional particle characteristics were evaluated numerically. For each particle, the volume (V) was geometrically calculated using the radii and coordinates of the particles and phase A elements. The particle volume comprising a proportion x of phase A domain was defined as $V(x)$. Perfectly liberated particle volumes were defined as V_A^{lib} and V_B^{lib} for phases A and B, respectively. The 3D information was obtained for particles featuring a height (H) ranging between 2 and 14.

2D characteristics were determined for sample sections with H ranging between 2 and 14 in a step of two. The particle sectional area (S) was geometrically calculated using the radii and coordinates of the particles and the phase A elements as well as H. The particle sectional area composed of a proportion x of phase A domain was defined as $S(x)$, whereas the perfectly liberated particle areas were defined as S_A^{lib} and S_B^{lib} for phases A and B, respectively.

2.3. Liberation distribution assessments

Fig. 2 conceptually shows the liberation distribution, which is a cumulative fraction of total volume (or area) of particles comprising a proportion x of phase A domains and the total volume (or area) of particles. Right and left ends of the distribution correspond to the degrees of liberation of phases A and B, respectively. In this study, the phase A content was divided into 12 classes, in

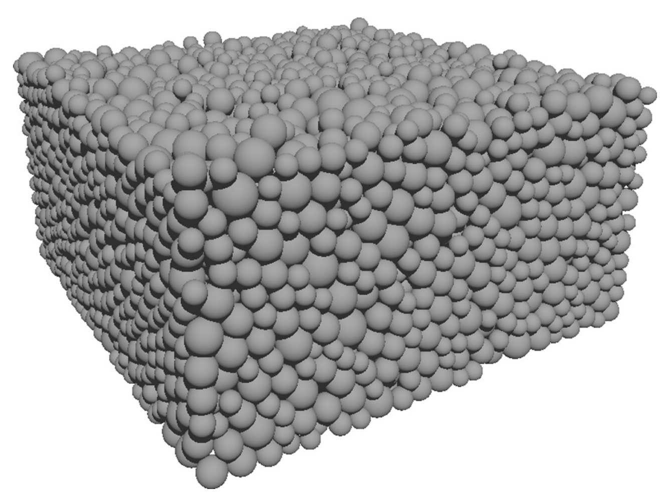

Fig. 1. Spherical particles packed in a cuboid using DEM.

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